Inferring Heap Abstraction Grammars

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July 30, 2012

Model Checking on Unbounded Structures

Model Checking

Verification by exploration of states

Input: int i_1, int i_2
while $i_1 \neq 0$ **do**
$i_1 := i_1 - 1$
$i_2 := i_2 + 1$
end while

Finite number of states ✓

Input: List l
Element $e := l.first()$
while $e \neq null$ **do**
$e := l.next(e)$
end while

Infinite number of states χ

Idea [Heinen et al., 2009]

Heap Abstraction Grammars ⇒ Finitely many heap states

Alphabets

Traditional Case

~~An alphabet is a finite set of symbols~~

Definition

An alphabet is a triple: $\Sigma := (N, T, rk)$,

where

- N – nonterminal symbols
- T – terminal symbols
- rk: $N \cup T \to \mathbb{N}$ – ranking function

and $N \cap T = \emptyset$

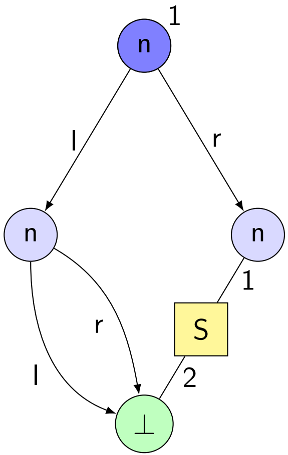
Heap Configurations

Definition

A heap configuration over an alphabet Σ and a finite set of symbols Γ is a tuple

$$G := (V, E, labV, labE, att, ext, \bot)$$

- V – nodes
- E – edges
- labV: $V \to \Gamma$ – node labels
- labE: $E \to N \cup T$ – edge labels
- att: $E \to V^*$ – attachment
- ext $\in V^*$ – external nodes
- $\bot \in V$ – null node



Heap Configurations (cont.)

Definitions

- Rank of an edge := number of nodes attached
- Terminal edge $:\Leftrightarrow labE(e) \in T$
- Terminal heap configuration $:\Leftrightarrow$ all edges are terminal

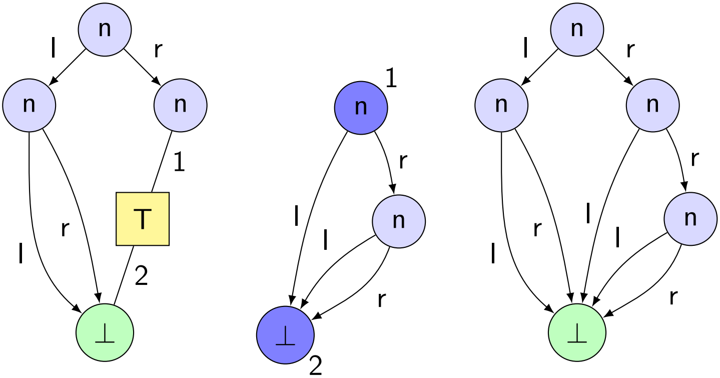
Requirements

- Terminal edges are of rank 2
- Rank of a label determines rank of edges
- All outgoing pointers of a class are represented
- All pointers are connected according to their type

Hyperedge Substitution

Given: heap configurations G, H ,edge e
Assumed: $e \in E_G$, $labE(e) \in N$, $rk(e) = |ext_H|$.

Substitute e with H, use external nodes as attachment points.



Data Structure Grammars

Definition

A Data Structure Grammar (DSG) I over an alphabet Σ and a set of symbols Γ is a tuple (N, T, P, S),

where

- N – nonterminal symbols
- T – terminal symbols
- $P = \{X_1 \to G_1, \ldots, X_n \to G_n\}$ – production rules
- S – axiom

$L(I) :=$ Set of all terminal configurations that can be derived from S

Heap Abstraction Grammar

Definition

A Heap Abstraction Grammar (HAG) is a Data Structure Grammar (DSG), that

1. Produces only valid heap configurations
2. Fulfills other restrictions [Heinen et al., tbp]

Any DSG that fulfils the first condition, but not the second one can be transformed into a HAG [Jansen, 2010].

Problem

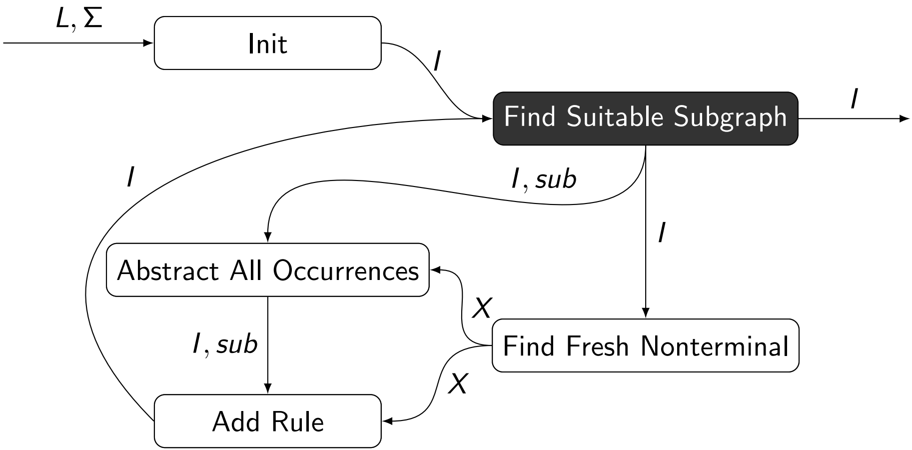
Given: Set of Heap Configurations L

Goal: Heap Abstraction Grammar I with $L(I) \supseteq L$

Overview

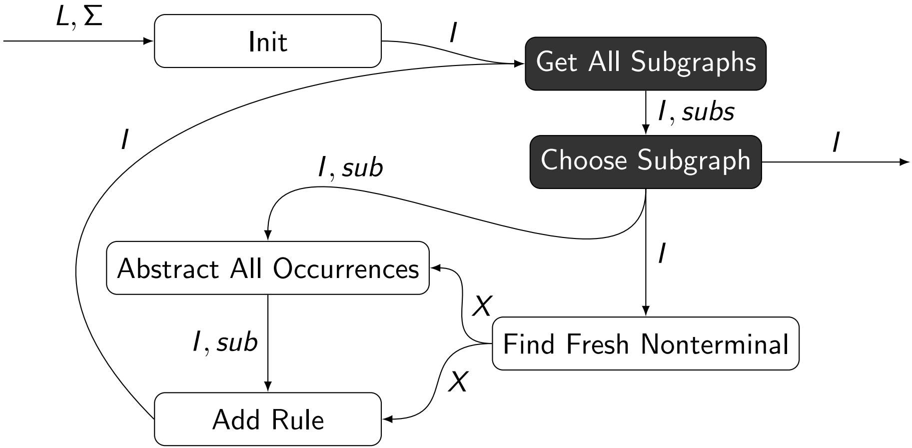
First: $L(I) = L$

Overview



L, Σ
Init
I
Find Suitable Subgraph
I
I
I, sub
I
Abstract All Occurrences
X
Find Fresh Nonterminal
I, sub
X
Add Rule

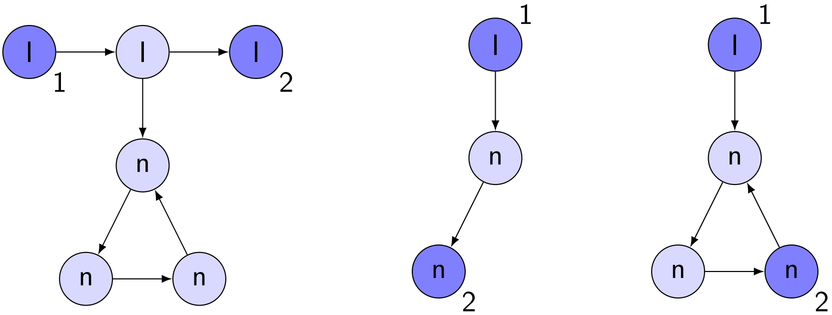
Overview


L, Σ
Init
I
Get All Subgraphs
I, subs
Choose Subgraph
I
I
I, sub
Abstract All Occurrences
I
X
Find Fresh Nonterminal
I, sub
X
Add Rule

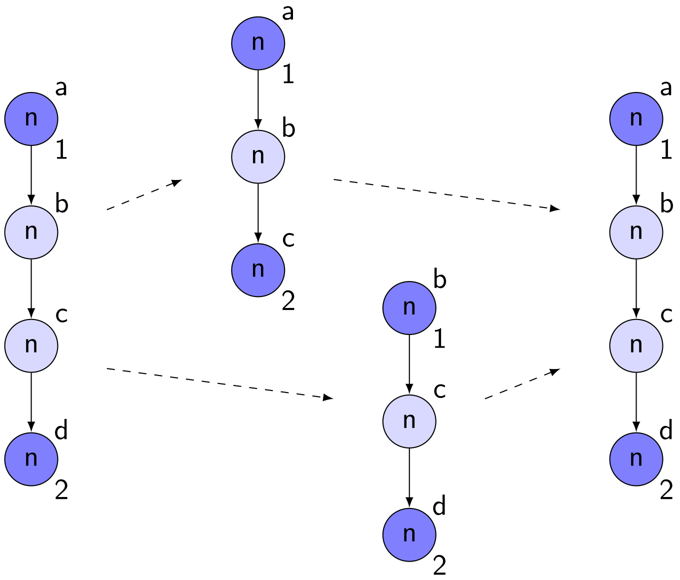
Subgraph Enumeration

Problem: Exponentially many subgraphs

Solution: Growing subgraphs [Jonyer et al., 2002]



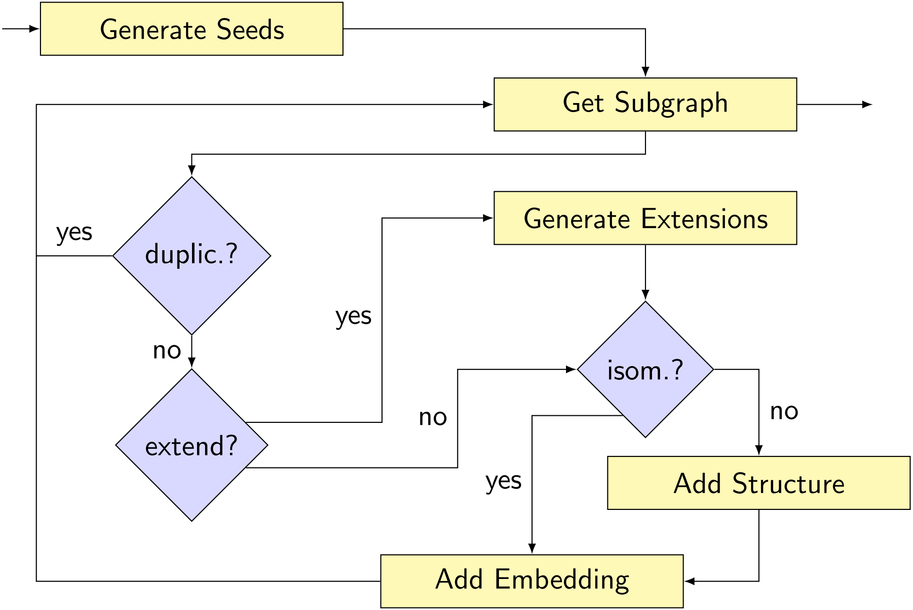
Duplicate and Isomorphic Subgraphs



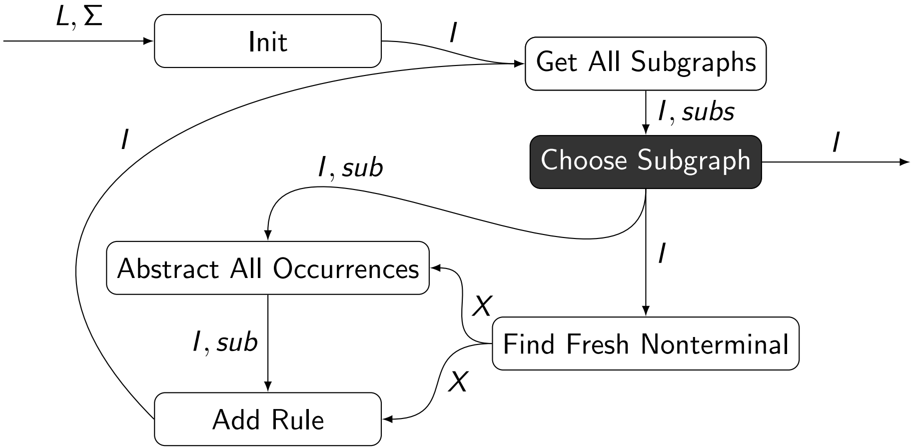
Duplication vs. Isomorphism

Isomorphic subgraphs $\hat{=}$ Same structure at different points in the graph

Duplicate subgraphs $\hat{=}$ Same structure at same point in the graph


Generate Seeds
Get Subgraph
yes
duplic.?
no
Generate Extensions
yes
extend?
no
isom.?
no
yes
Add Structure
Add Embedding

Overview



L, Σ
Init
I
Get All Subgraphs
$I, subs$
Choose Subgraph
I
I
I, sub
Abstract All Occurrences
I
X
Find Fresh Nonterminal
I, sub
X
Add Rule

Choosing a Subgraph – Minimum Description Length

Question: Which subgraph to abstract?

Answer: Optimization of cost function

Minimum Description Length [Rissanen, 1978]

$$sub = \operatorname{argmin}(cost(X \to H) + cost(L')),$$

where L' is L under the assumption that the rule $X \to H$ is known.

Description Length

Definition

Cost of a heap configuration G:

$$cost(G) = |V| + |E| + \sum_{e \in E} rk(e)$$

Cost of a set of heap configurations L:

$$cost(L) = \sum_{G \in L} cost(G)$$

Description length of a production rule:

$$cost(X \to H) = 1 + cost(H)$$

Several definitions possible

Simplification

After a single substitution of H

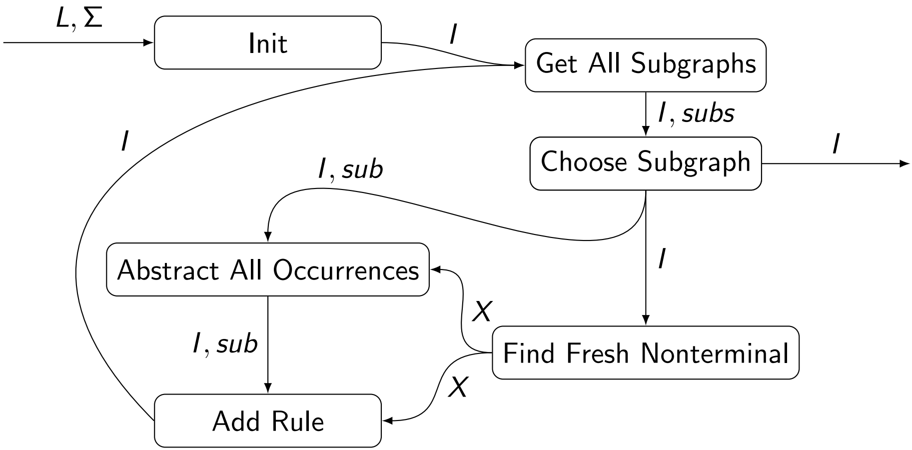
$$cost(L') = cost(L) - cost(H) + 2 \cdot |ext_H| + 1$$

After n substitutions of H

$$cost(L') = cost(L) - n \cdot (cost(H) + 2 \cdot |ext_H| + 1)$$

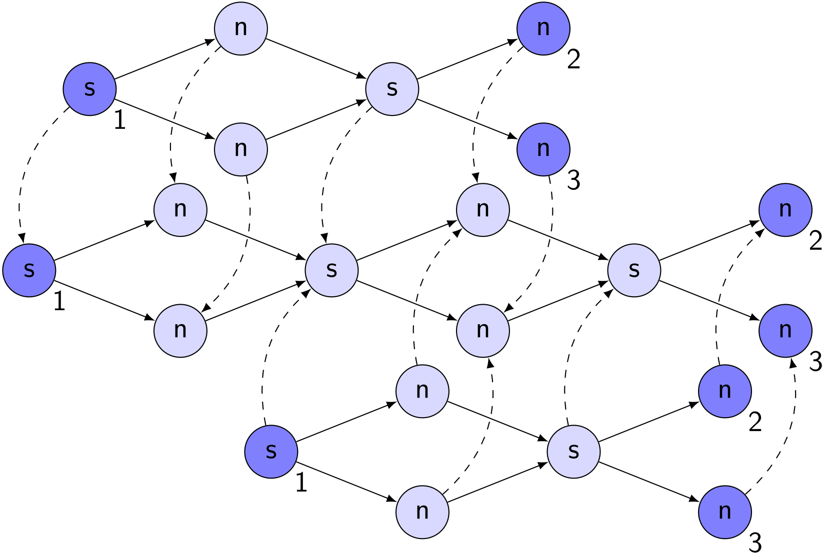
$\Rightarrow$ Computation of cost without actual replacement

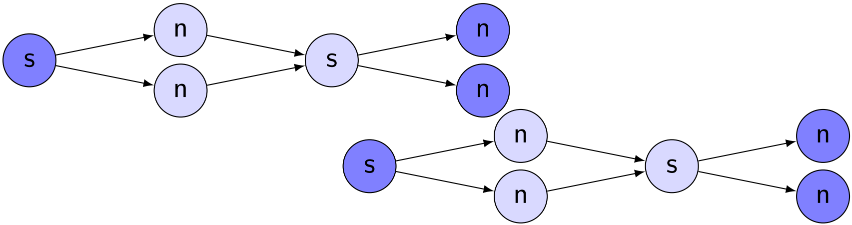
Overview



Now: $L(I) \supseteq L$

Recursive Structures [Jonyer et al., 2002]





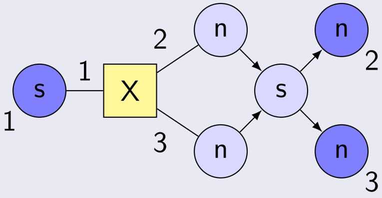
Definition

A structure S is recursive, iff there exist two embeddings, such that

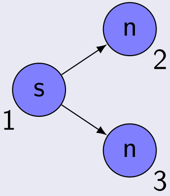
- There are no nonterminal edges attached to the external nodes
- The two embeddings do not share internal nodes
- The external nodes can be partitioned into entry- and exit-nodes
- There are no incoming pointers to the entry-nodes
- There are no outgoing pointers from the exit-nodes
- The exit-nodes of one embedding can be reached from the entry-nodes of the other one

Recursive Rules added to the Grammar

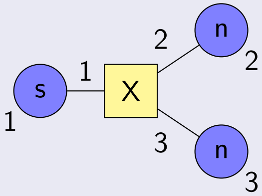
Concatenation Rules



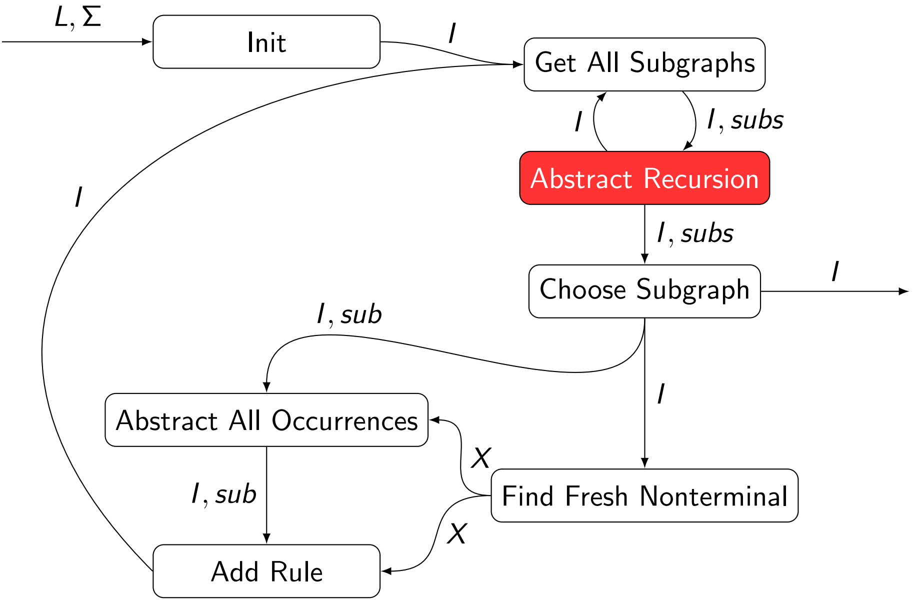
Finalization Rule



Abstraction in Graph



Complete Algorithm



Singly Linked List

Input: Singly linked lists with 25 to 200 nodes

Nodes	Subgraphs [ms]	Complete [ms]
25	90	102
50	285	305
75	437	500
100	642	682
125	1 001	1 040
150	1 455	1 526
175	1 884	2 000
200	2 895	3 028

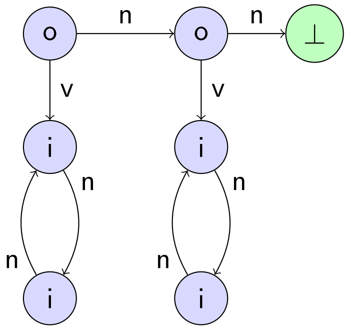
Singly Linked Circular List

Input: Singly linked circular lists with 25 to 200 nodes

Nodes	Subgraphs [ms]	Complete [ms]
25	75	90
50	261	281
75	451	464
100	680	658
125	979	1 032
150	1 465	1 511
175	1 889	1 933
200	2 805	2 995

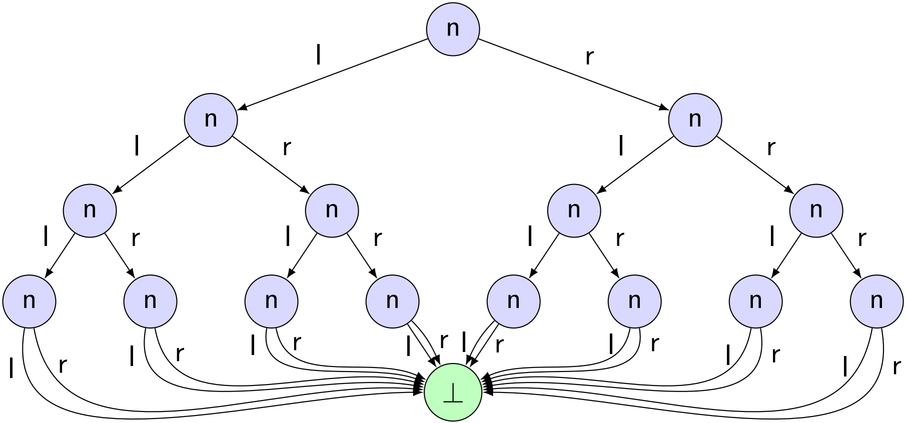
Singly Linked Nested List

Input: Singly linked nested lists

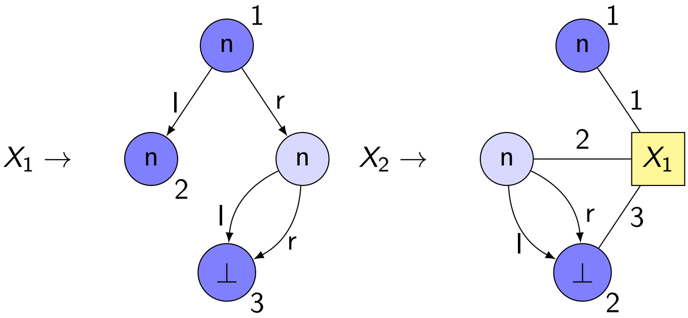


Outer	Inner	Inner [ms]	Outer [ms]
2	2	9	8
2	4	19	5
4	2	31	16
4	4	394	11
4	5	1 280	20
5	2	96	23
5	4	2 701	22
5	5	51 608	23

Binary Tree



Binary Tree – Rules



Summary

- Definition: Heap Configurations and Heap Abstraction Grammars
- Problem: Generate grammar from set of Heap Configurations
- Solution: Abstract subgraphs step by step
  1. Find recursive subgraphs
  2. Abstract subgraph with greatest gain in description length
- Backed by experimental results

Further Research

- Subgraph enumeration avoids certain graphs
  - Efficiently enumerate subgraphs with unconnected internal nodes
- Recursion only works for simply concatenations
  - Allow multiple sets of entry- and exit-points

Thank you for your attention

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