Verifying the Heap

Original Research: [Rey02]

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Motivation

Up until now: Each variable holds a single value from $\mathbb{N}$

Most prominent missing feature: References

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Most prominent missing feature: References

Excludes lots of interesting concepts: Lists, Trees, Graphs, OOP

Solution: Introduce formal handling of heap

Outline

Notation

Partial functions	$f : A \rightharpoonup B$
Undefined point	$f(x) = \perp$
Evaluation	$[\![c]\!]_s$ $[\![e]\!]_s$
Arbitrary Value	$f : x \mapsto$ -

Using the Heap

Four primitives:

- Allocation
- Lookup
- Mutation
- Deallocation

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Allocation	$x := \textbf{alloc}(e_1, \ldots, e_n)$
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Allocation	$x := \textbf{alloc}(e_1, \ldots, e_n)$
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Mutation	$[e_1] := e_2$
Deallocation	$\textbf{free}(e)$

New Language Obj

$\mathit{comm} :=$

New Language OBJ

$$\begin{array}{rcll} comm := & & \textbf{skip} \quad \mid \quad x := expr_a & \\ & \mid & comm;comm & \\ & \mid & \textbf{if}\ expr_b\ \textbf{then}\ comm\ \textbf{then}\ comm & \text{IMP} \\ & \mid & \textbf{while}\ expr_b\ \textbf{do}\ comm & \end{array}$$

$$\begin{array}{rlll} expr_a := & expr_a + expr_a & \mid \quad \ldots & \text{IMP} \\ expr_b := & expr_b \wedge expr_b & \mid \quad \ldots & \end{array}$$

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$$
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\mathit{comm} := & & \textbf{skip} \quad | \quad x := \mathit{expr}_a & \\
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& | & \textbf{while } \mathit{expr}_b \textbf{ do } \mathit{comm} & \\
& & & \\
& | & x := \textbf{alloc}(\mathit{expr}_a, \ldots, \mathit{expr}_a) & \\
& | & x := [\mathit{expr}_a] \quad | \quad [\mathit{expr}_a] := \mathit{expr}_a & \textsc{Obj} \\
& | & \textbf{free}(\mathit{expr}_a) & \\
& & & \\
\mathit{expr}_a := & & \mathit{expr}_a + \mathit{expr}_a \quad | \quad \ldots & \\
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$$

Introducing the Heap (Intuition)

Stack

Heap

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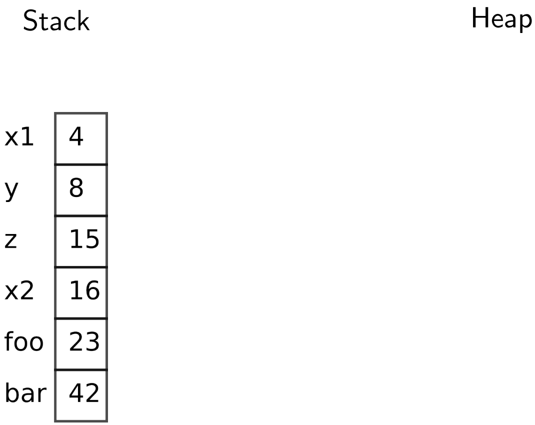

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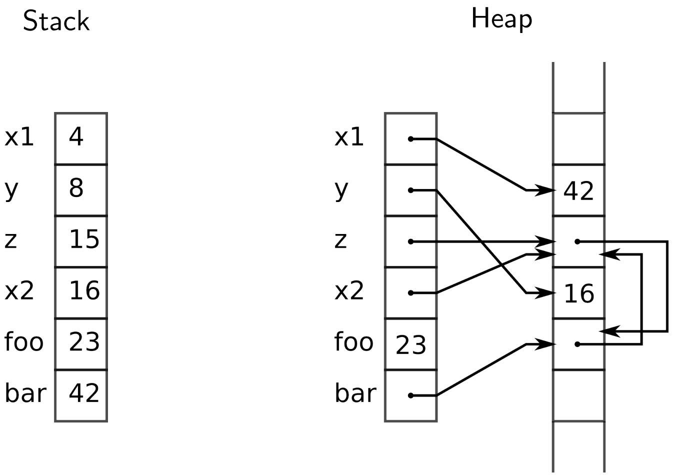

Observations

- Still a fixed (finite) set of variables: *Var*
- Variables can hold *Atom*s or *Add*resses
- *Add*resses point to either *Atom*s or *Add*resses

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Idea: Function from variables to heap,
Function from heap to values

Introducing the Heap (formally)

Set of variables	Var	
Set of atoms	Atom	
Set of addresses	Add	
Set of values	$\mathit{Atom} \cup \mathit{Add}$	$=: \mathit{Val}$
	$\mathit{Atom} \cap \mathit{Add}$	$\stackrel{!}{=} \varnothing$

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Stack	$s :$	$Var \rightharpoonup Val$
Heap	$h :$	$Add \rightharpoonup Val$

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New configuration: $\langle s, h \rangle$

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Stack	$s :$	$Var \rightharpoonup Val$
Heap	$h :$	$Add \rightharpoonup Val$

New configuration: $\langle s, h \rangle$

New execution relation: $(\langle s, h \rangle, c) \Downarrow \langle s', h' \rangle$

Address arithmetic

Two more constraints:

- Atoms should still be integers
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$$\Rightarrow \mathit{Atom} \subsetneq \mathbb{N}, \mathit{Add} \subsetneq \mathbb{N}$$

Inference Rules

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Where $\quad s' = s[x/a] \quad$ and $\quad h' = h[(a+0)/v_1]\ldots[(a+n-1)/v_n]$

More Motivation

We have: Operational semantics of OBJ

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We want: Verification of OBJ programs

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Solution: Extend axiomatic semantics of of IMP

Reminder Hoare Calculus [Hoa69]

Judgments of the form $\{Pre\}\, c\, \{Post\}$

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Pre, $Post$:	Logical formulas over $\mathbb{N}$, Var
	depends on underlying theory

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New underlying theory: Separation Logic

Separation Logic

Four new operators:

Empty Heap	**emp**
Singleton Heap	$\cdot \mapsto \cdot$
Separating Conjunction	$\cdot * \cdot$
Separating Implication	$\cdot \mathrel{-\!\!*} \cdot$

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Example

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To show:

$$\frac{}{\{x \mapsto 23 * y \mapsto 15\}\ \mathit{free}(x)\ \{y \mapsto 15\}}$$

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Framing rule [Rey02]

$$\text{frame}\,\frac{\{p\}\; c\; \{q\}}{\{p * r\}\; c\; \{q * r\}}\,,$$

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"Consequence rule of Separation Logic"

Example, take 2

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$$\text{cons}\ \dfrac{\text{frame}\ \dfrac{}{\{x \mapsto 23 * y \mapsto 15\}\ \mathit{free}(x)\ \{\mathbf{emp} * y \mapsto 15\}}}{\{x \mapsto 23 * y \mapsto 15\}\ \mathit{free}(x)\ \{y \mapsto 15\}}$$

Example, take 2

$$\dfrac{\dfrac{\dfrac{}{\{x \mapsto 23\}\ \mathit{free}(x)\ \{\mathbf{emp}\}}\ \text{free}}{\{x \mapsto 23 * y \mapsto 15\}\ \mathit{free}(x)\ \{\mathbf{emp} * y \mapsto 15\}}\ \text{frame}}{\{x \mapsto 23 * y \mapsto 15\}\ \mathit{free}(x)\ \{y \mapsto 15\}}\ \text{cons}$$

What now?

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- Verify programs using the heap
  - e.g. Garbage Collector [Yan01]
- Shape analysis [DOY06]
- Prove information hiding of library [OYR04]

Information hiding [OYR04]

Previously: Program as single, monolithic procedure

Information hiding [OYR04]

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Now: Program separated into several functions, libraries

Libraries in Separation Logic

Procedure Names:	$k_1, \ldots, k_n$
Interface Specification:	$\{P_1\}k_1\{Q_1\}[X_1], \ldots, \{P_n\}k_n\{Q_n\}[X_n]$
Implementations:	$c_1, \ldots, c_n$
Resource Invariant	r
Internal Variables:	Y

Using Libraries in Separation Logic

We have to show:

$$\{P_i * r\}c_i\{Q_i * r\}, \text{ for each procedure } k_i$$

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Then we can show:

$$\{P\}c\{Q\}, \text{ for the main program}$$

using the assumptions:

$$\{P_i\}c_i\{Q_i\}, \text{ for all library procedures,}$$

if

$$c \text{ does not use variables in } r$$

Benefits

- Change of implementation: Just prove new implementation
- Wrong use of interface variables can be precluded

Downsides

- Definition of modules is very clunky
- Use of interface variables: Either full access or none

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- General idea: Introduce mathematical handling of heap
- Formally defined the heap
  - Syntactic definitions
  - Operational Semantics
  - Axiomatic Semantics (Hoare Calculus + Separation Logic)
- Example for usage of Axiomatic Semantics

Model Checking [Clarke, Emerson, Sifakis]

Idea: Explore state space

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Graph Grammars [HNR10]

	String Grammars	Graph Grammars
Description of:	Set of strings	Set of graphs
Atoms:	Characters	Objects of the heap
Derivation:	Replace Nonterminals	Replace inactive parts of heap

⇒ Finite description of all possible heap configurations

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⇒ Finite description of all possible heap configurations

⇒ Finite state space

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