Easy to Win, Hard to Master:
Optimal Strategies in Parity Games with Costs

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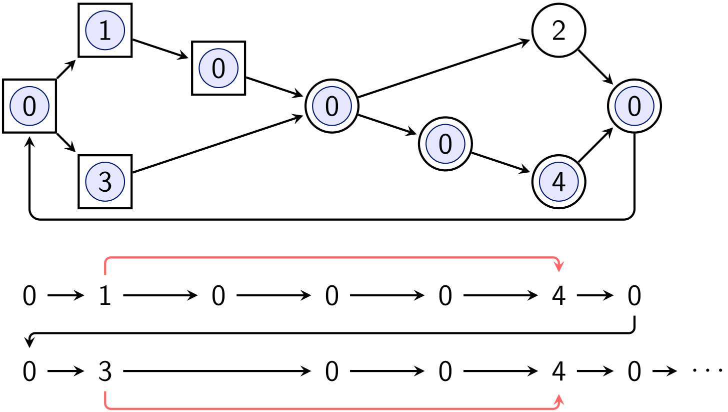
Parity Games

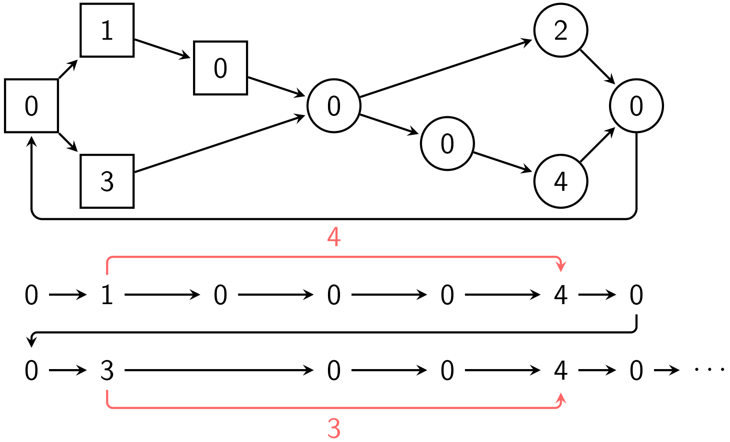


Deciding winner in $\mathrm{UP} \cap \textsc{co}\text{-}\mathrm{UP}$

Positional Strategies

Example due to (Fijalkow and Chatterjee, Infinite-state games, 2013)

Finitary Parity Games



Goal for Player 0: Bound response times

Example due to (Fijalkow and Chatterjee, Infinite-state games, 2013)

Decision Problem

Theorem (Chatterjee et al., Finitary Winning, 2009)

The following decision problem is in $\textsc{PTime}$*:*

Input: *Finitary parity game* $\mathcal{G} = (\mathcal{A}, \mathrm{FinParity}(\Omega))$

Question: *Does there exist a strategy* σ *with* $\mathrm{Cst}(\sigma) < \infty$*?*

Theorem

The following decision problem is $\textsc{PSpace}$*-complete:*

Input: *Finitary parity game* $\mathcal{G} = (\mathcal{A}, \mathrm{FinParity}(\Omega))$*,*
bound $b \in \mathbb{N}$

Question: *Does there exist a strategy* σ *with* $\mathrm{Cst}(\sigma) \leq b$*?*

From Finitary Parity to Parity

Given: Finitary parity game $\mathcal{G} = (\mathcal{A}, \mathrm{FinParity}(\Omega))$, bound $b \in \mathbb{N}$.

Lemma

Deciding if Player 0 has strategy σ with $\mathrm{Cst}(\sigma) \leq b$ is in $\mathrm{PS{\small PACE}}$.

Idea: Simulate game, keeping track of open requests.

Lemma

Player 0 has such a strategy iff she "survives" $p(|\mathcal{G}|)$ steps in extended game $\mathcal{G}'$.

Algorithm:

Simulate all plays in $\mathcal{G}'$ on-the-fly for $p(|\mathcal{G}|)$ steps using an alternating Turing machine.

$\Rightarrow$ Problem is in $\mathrm{APT{\small IME}}$

(Chandra et al., Alternation, 1981)

$\Rightarrow$ Problem is in $\mathrm{PS{\small PACE}}$

PSpace-completeness

Lemma

The given decision problem is in PSPACE*.*

Lemma

The given decision problem is PSPACE*-hard.*

Proof: By reduction from quantified Boolean formulas

$\Rightarrow$ The given decision problem is PSPACE-complete

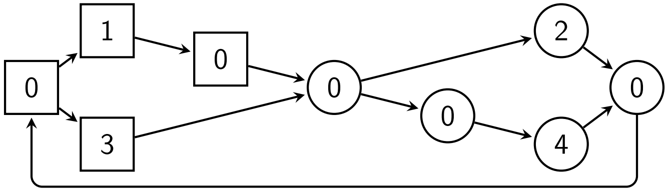
Memory Requirements (for Player 0)

Theorem

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSPACE-membership

Necessity:



For given parameter d:

- Generalize to d colors
- Repeat d times

Player 1 has d choices of d actions

$\Rightarrow$

Player 0 needs $\approx 2^d$ memory states

Conclusion

	Parity	Finitary Parity	
		Winning	Optimal
Complexity	$\mathrm{UP} \cap \mathrm{co}\text{-}\mathrm{UP}$	PTime	PSpace-comp.
Strategies	1	1	Exp.

Take-away: Forcing Player 0 to answer quickly in (finitary) parity games makes it harder

- to decide whether she can satisfy the bound
- for her to play the game