VLDL Satisfiability and Model Checking via Tree Automata

Alexander Weinert

Saarland University

December 12th, 2017

FSTTCS 2017 - Kanpur, India

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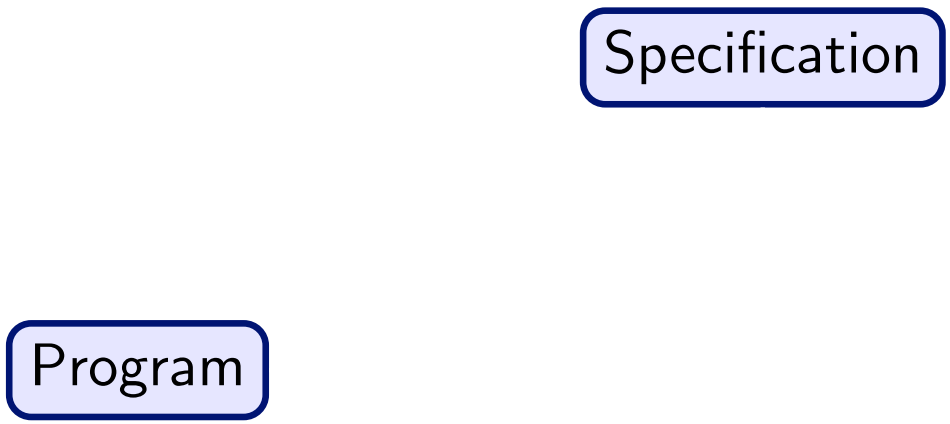

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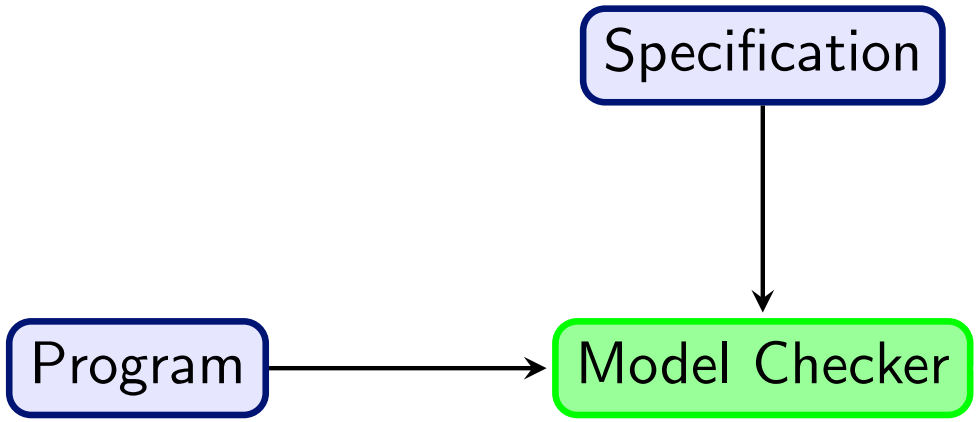

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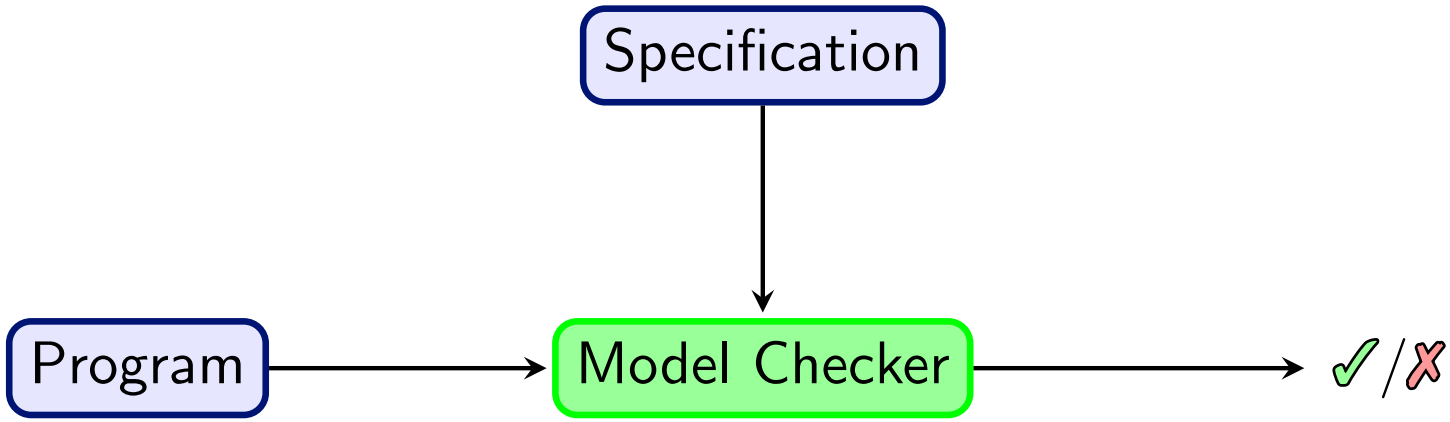

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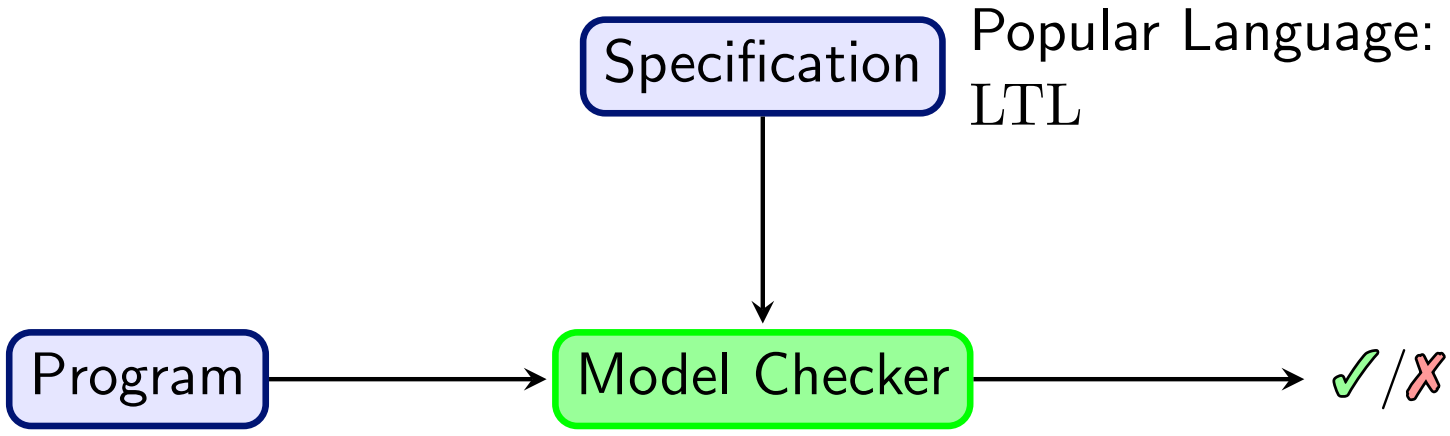

An Example

```
def f():
  if(/*...*/):
    cd("folder")
  else:
    cd("..")

def main():
  /*...*/
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Not expressible in LTL!

Visibly Linear Dynamic Logic (VLDL)[1]

Syntax of VLDL:

$$\varphi = p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \langle\mathfrak{A}\rangle\varphi \mid [\mathfrak{A}]\varphi$$

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$\langle\mathfrak{A}\rangle\varphi$

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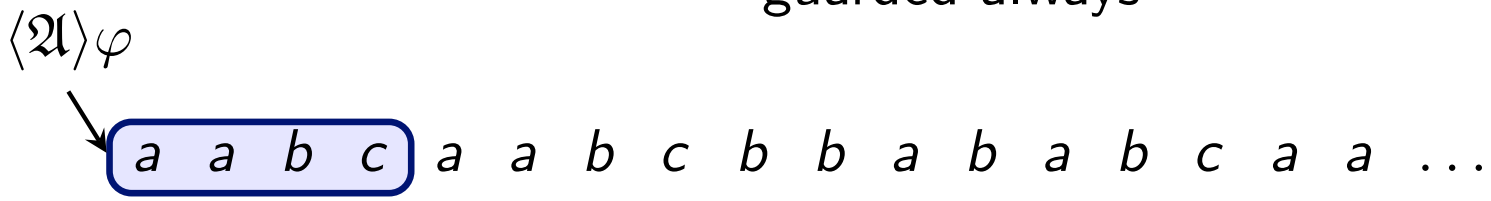

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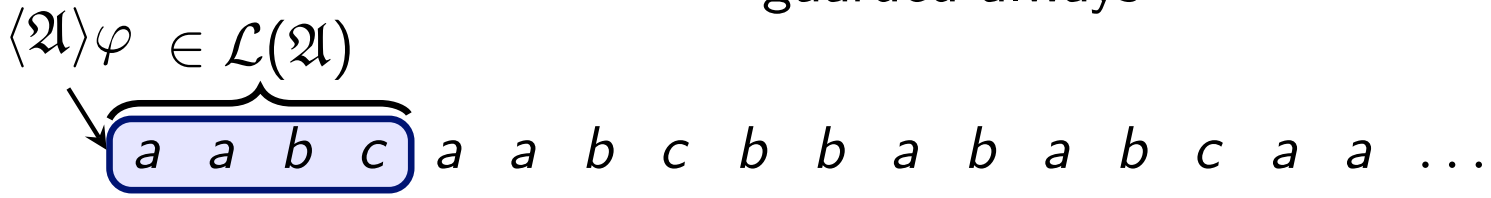

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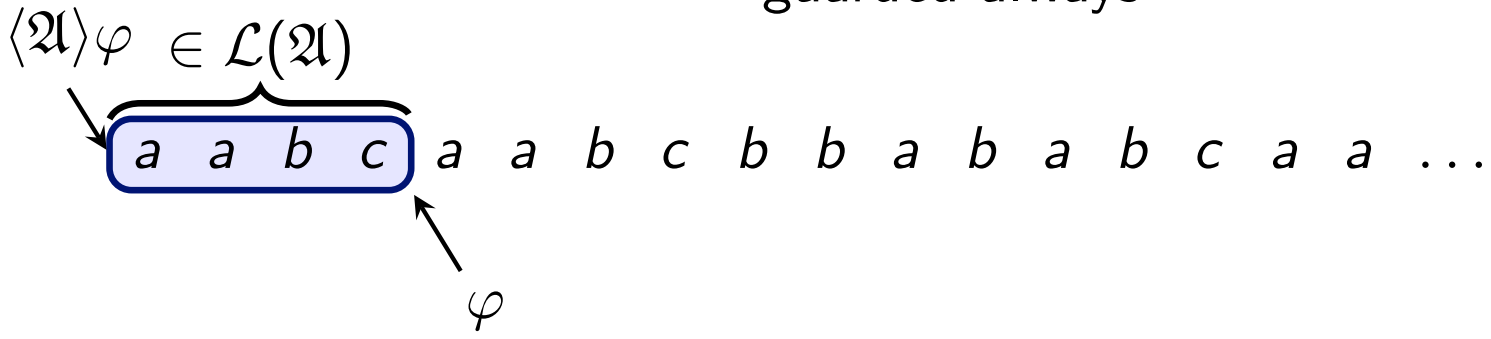

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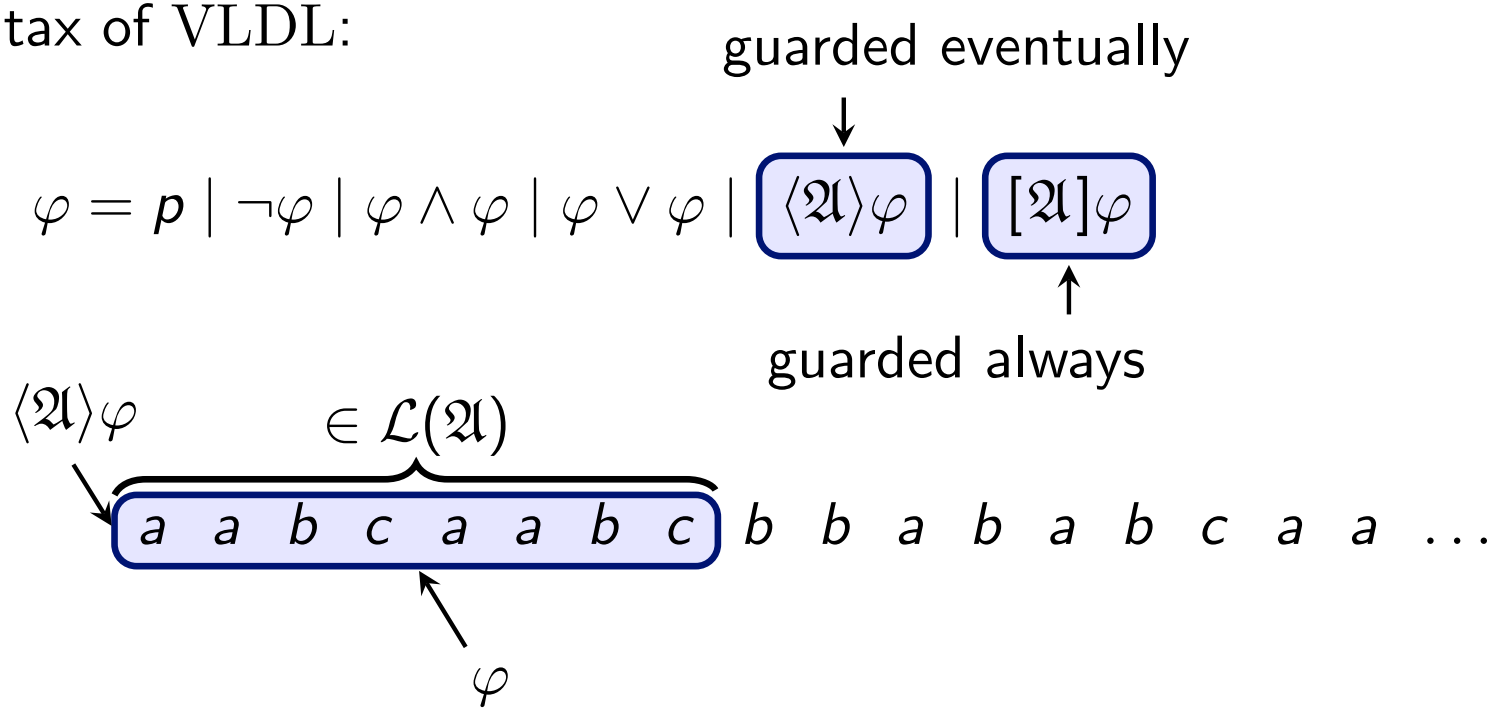

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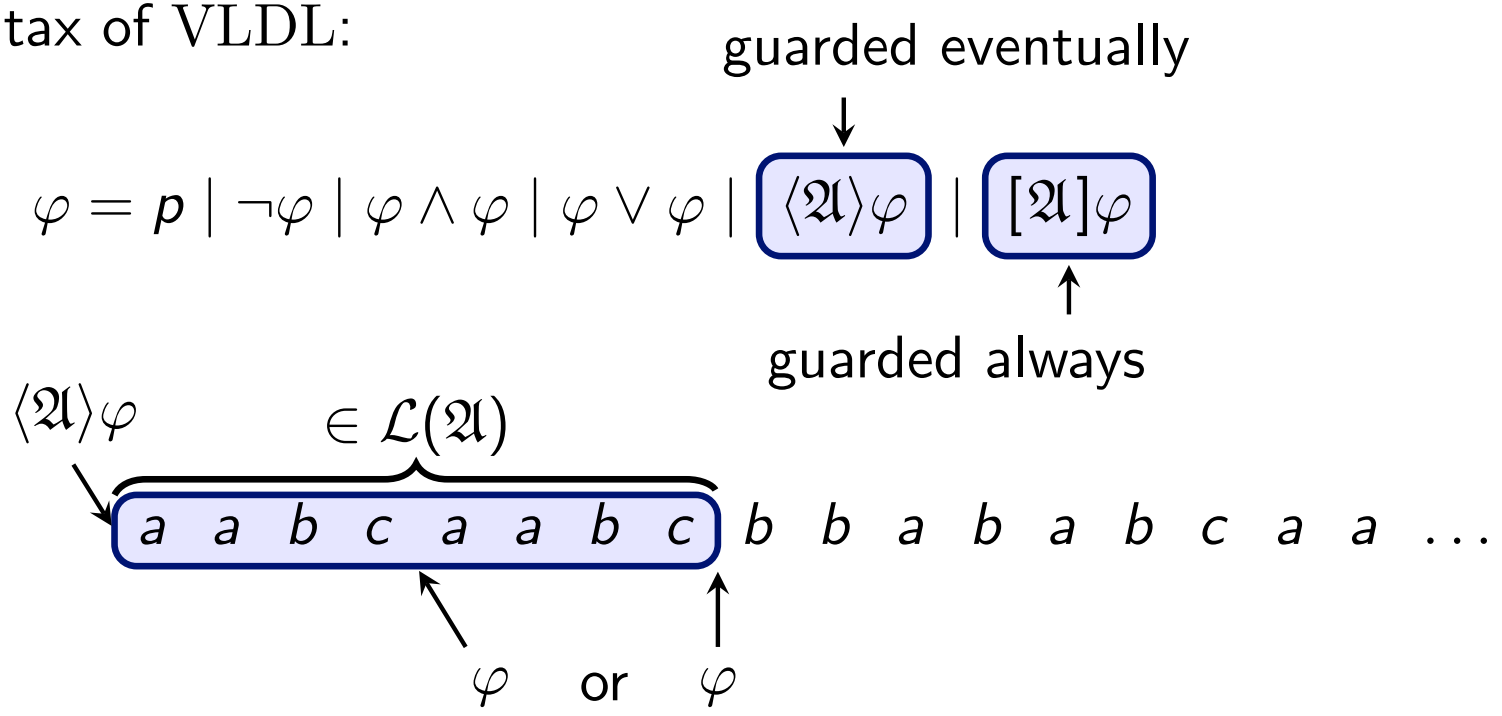

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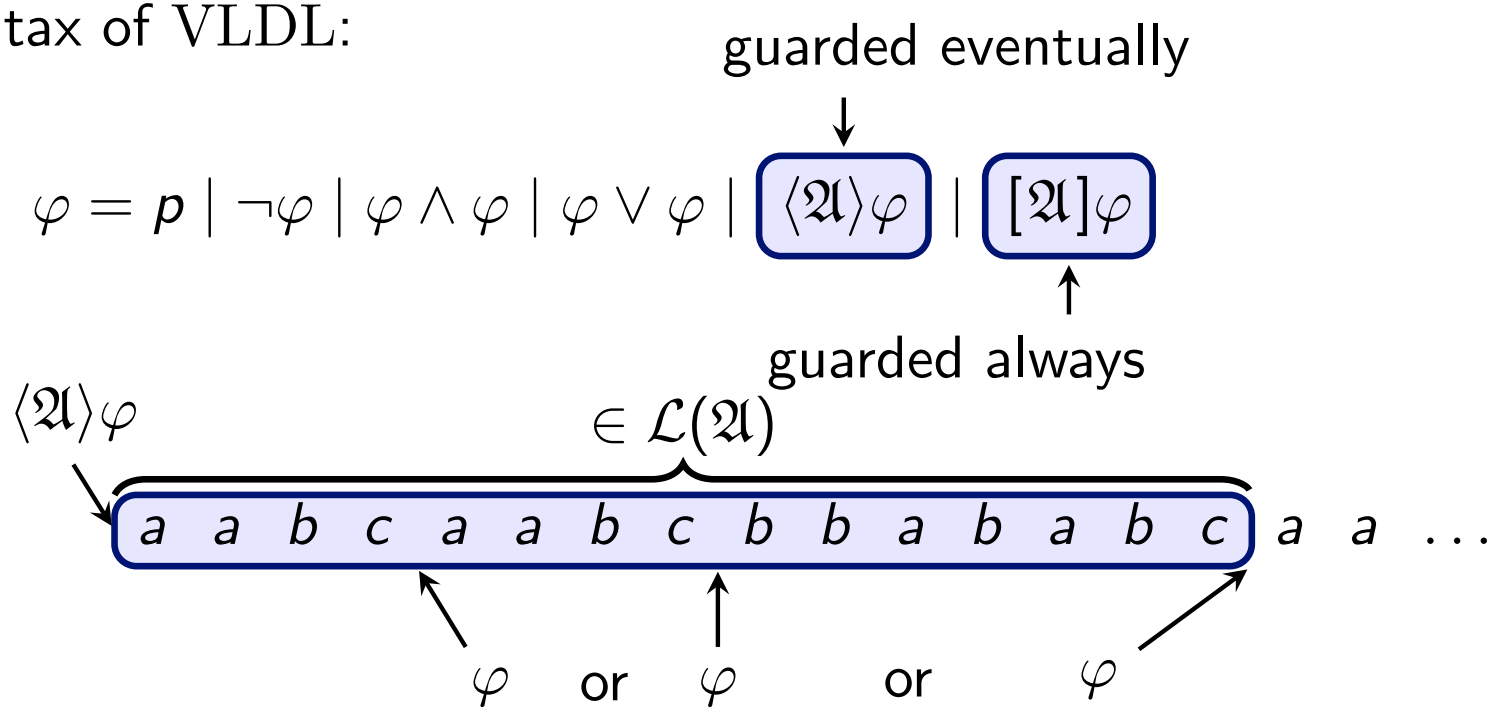

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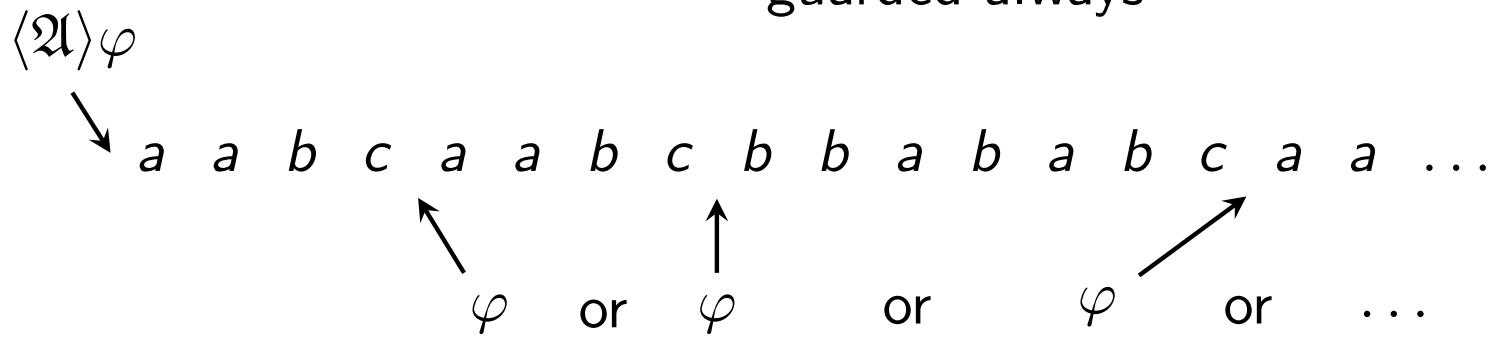

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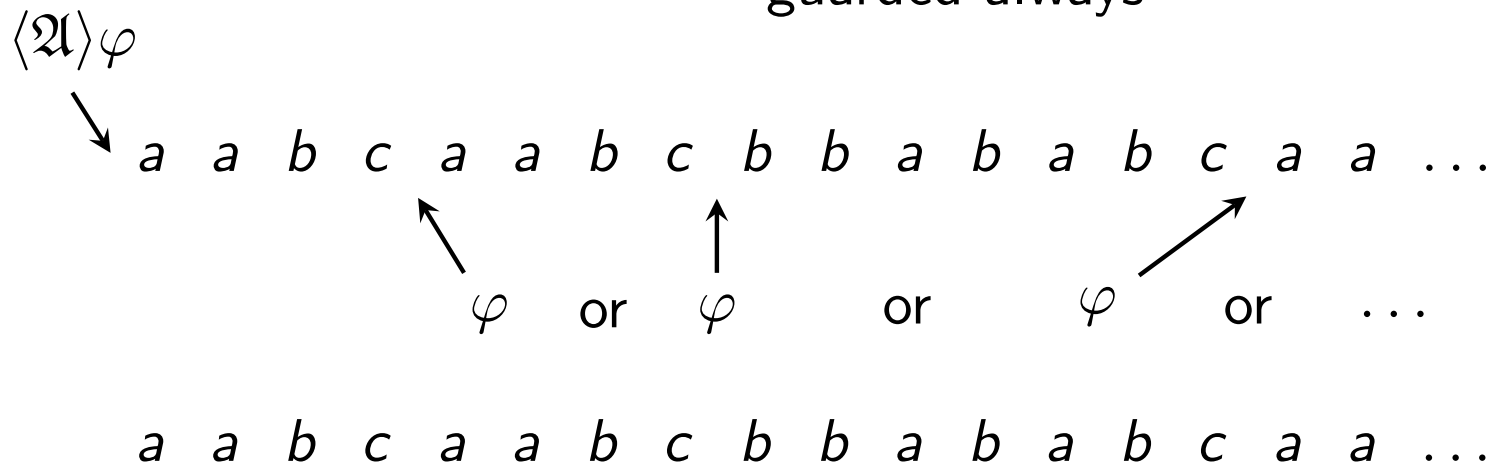

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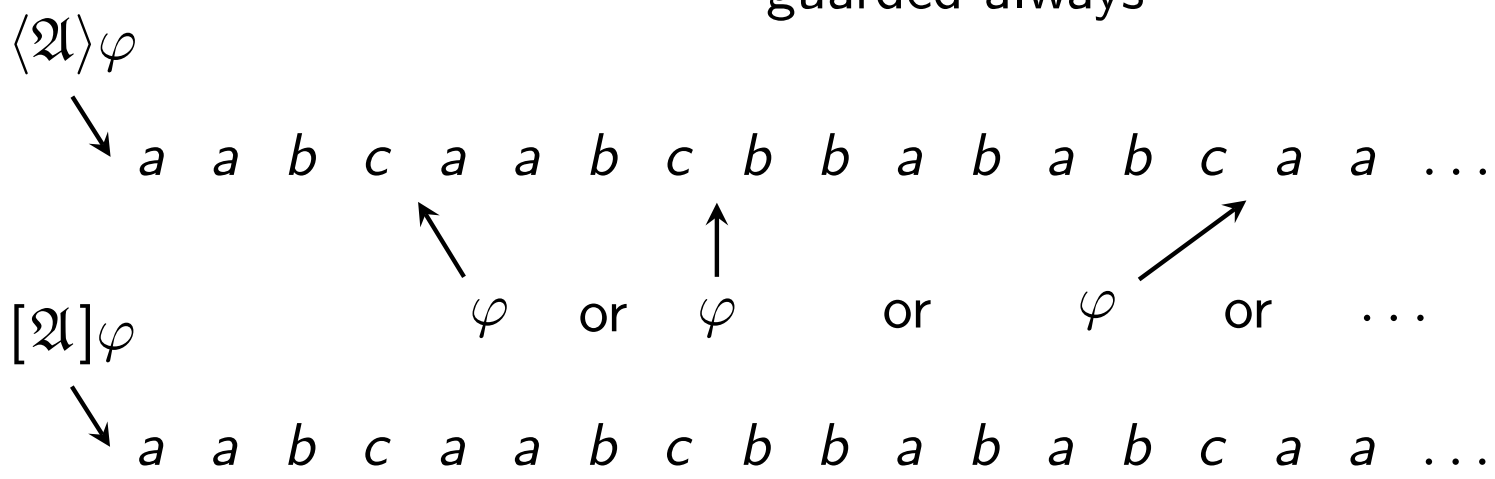

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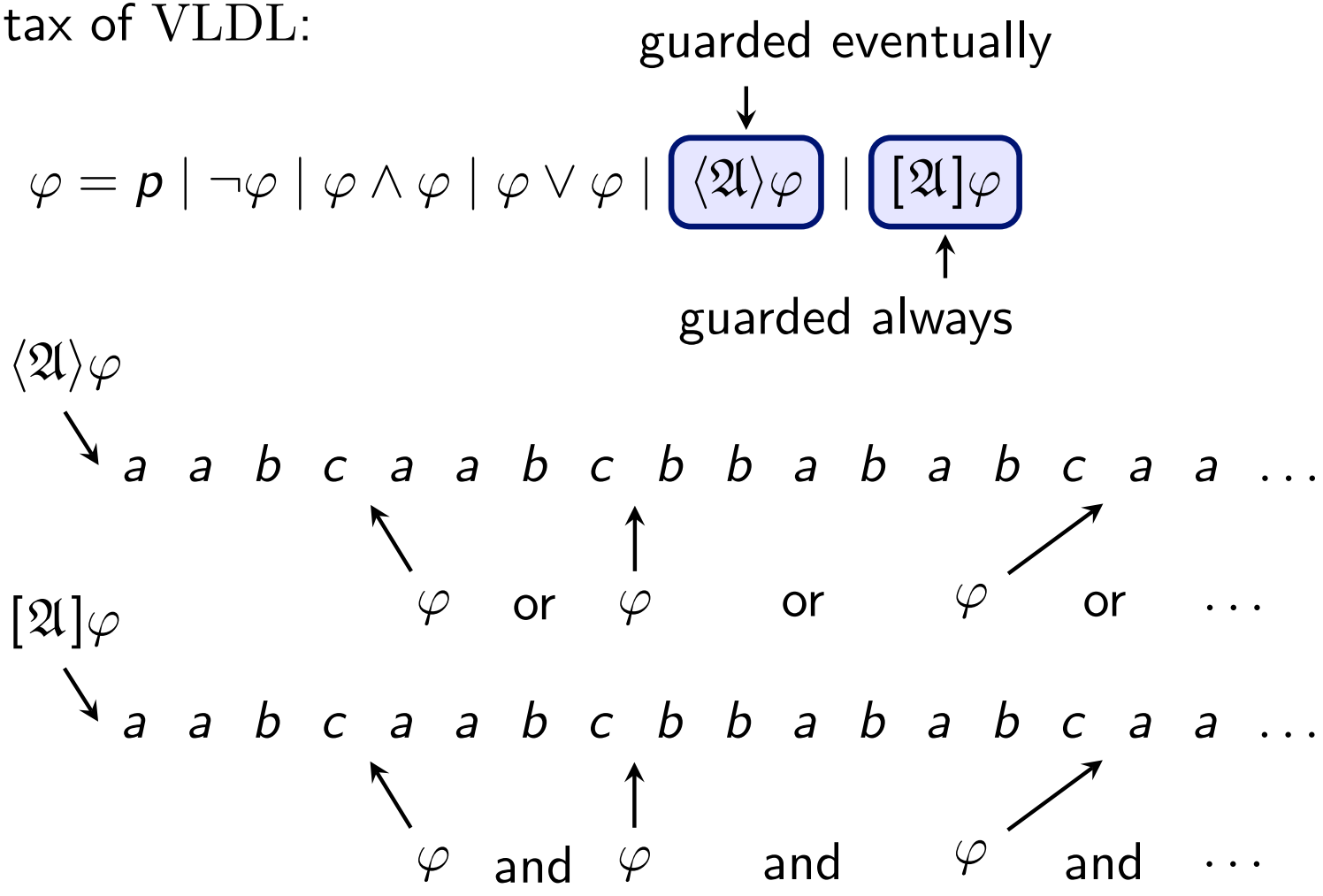

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Visibly Pushdown Automata[2]

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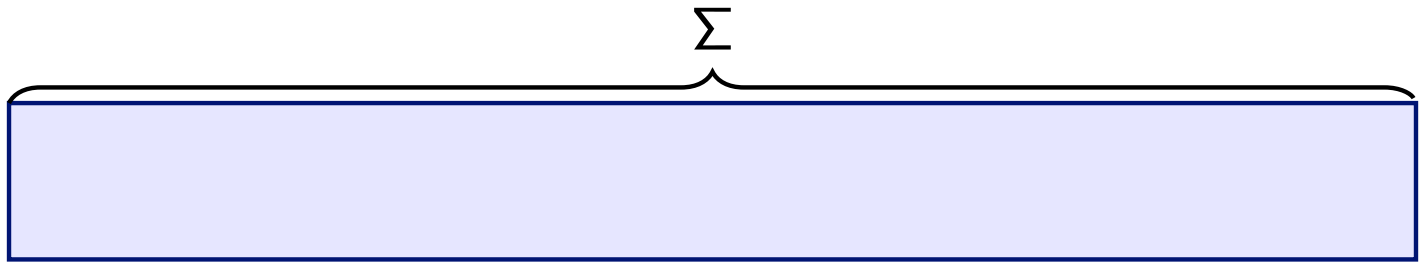

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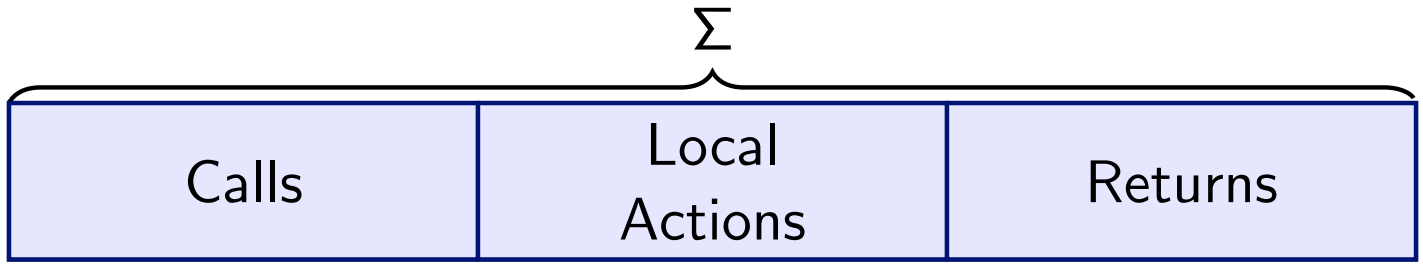

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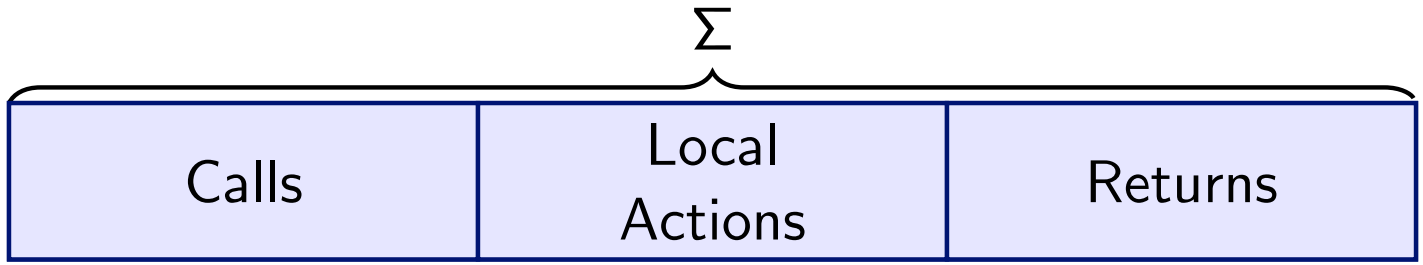

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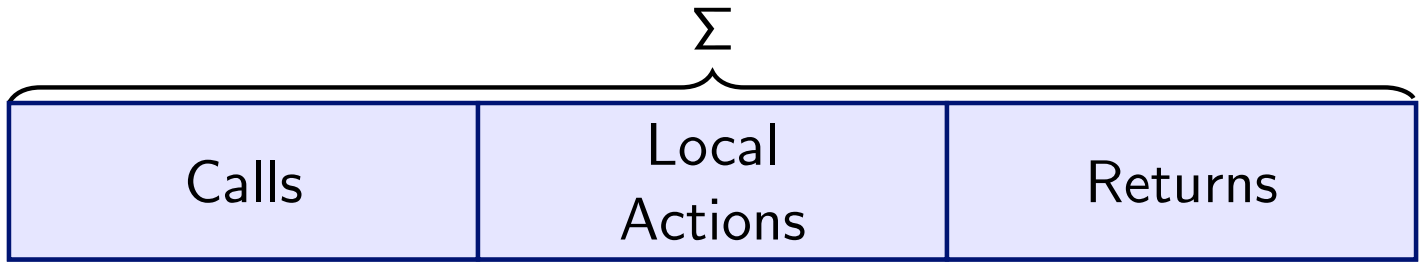

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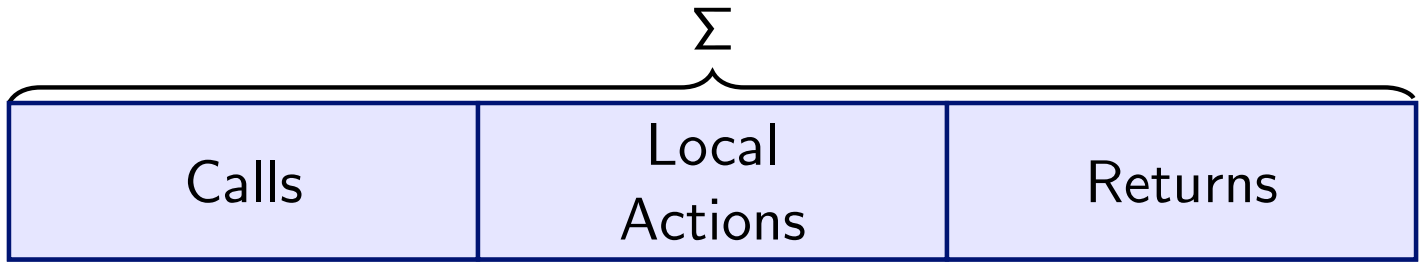

- When reading call, automaton has to push onto the stack
- When reading return, automaton has to pop off the stack
- When reading local action, automaton has to ignore the stack

$\Rightarrow$ Closed under intersection!

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VLDL Complexity[3]

VLDL: Extension of LTL,
temporal operators guarded by visibly pushdown automata

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Satisfiability	ExpTime-complete

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Contribution:
Novel, conceptually simple algorithms

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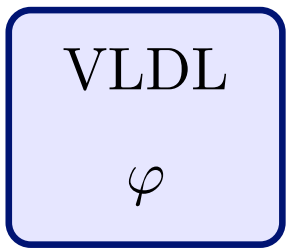
VLDL Satisfiability and Model Checking

Theorem (W. and Zimmermann, 2016)

VLDL *Satisfiability is* ExpTime*-complete.*



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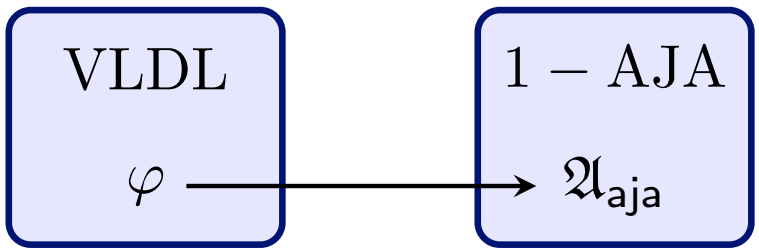

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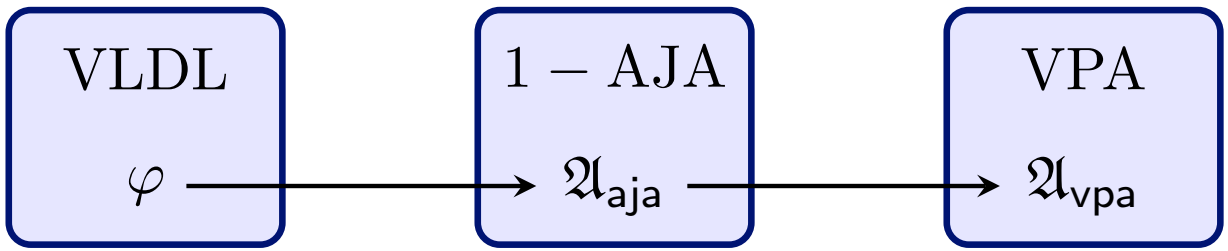

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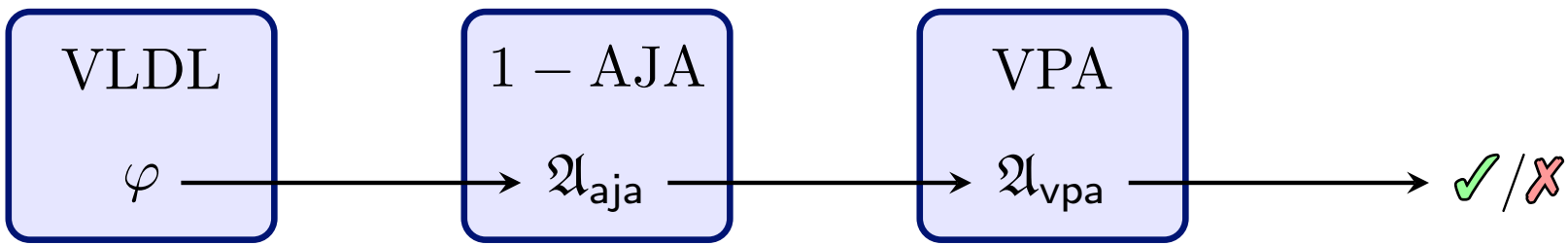

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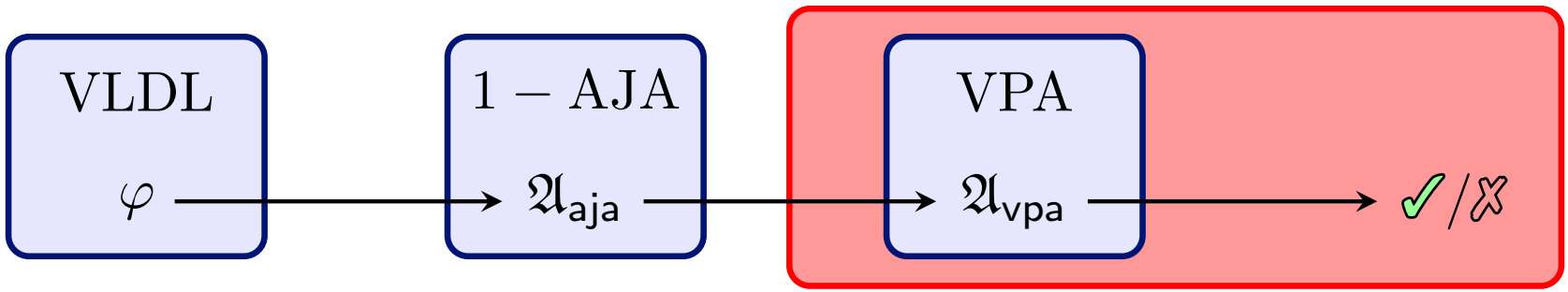

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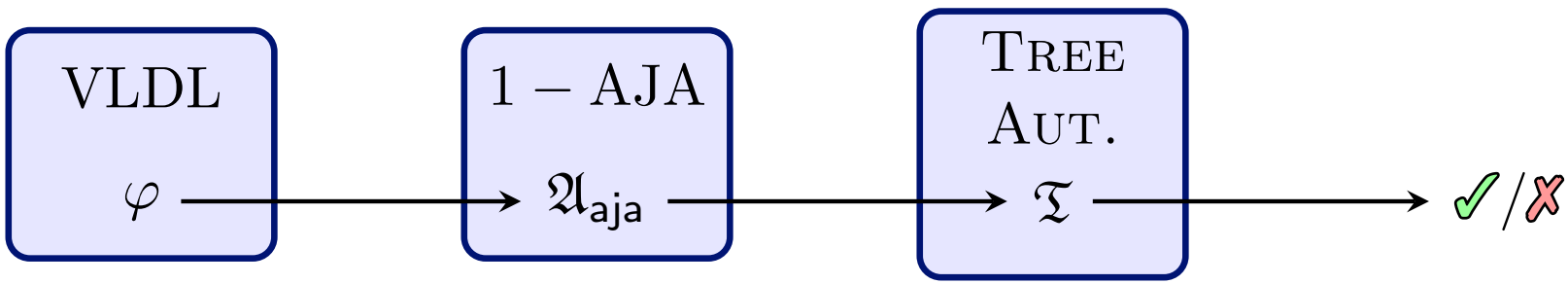

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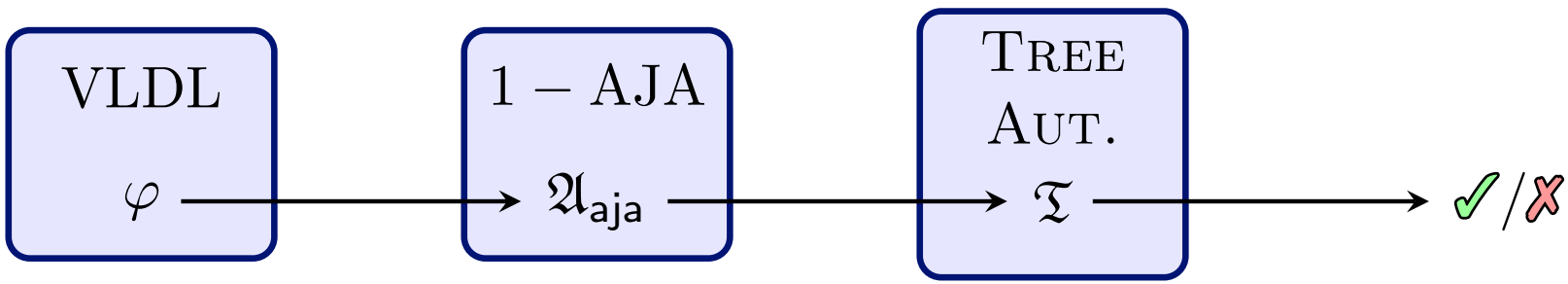

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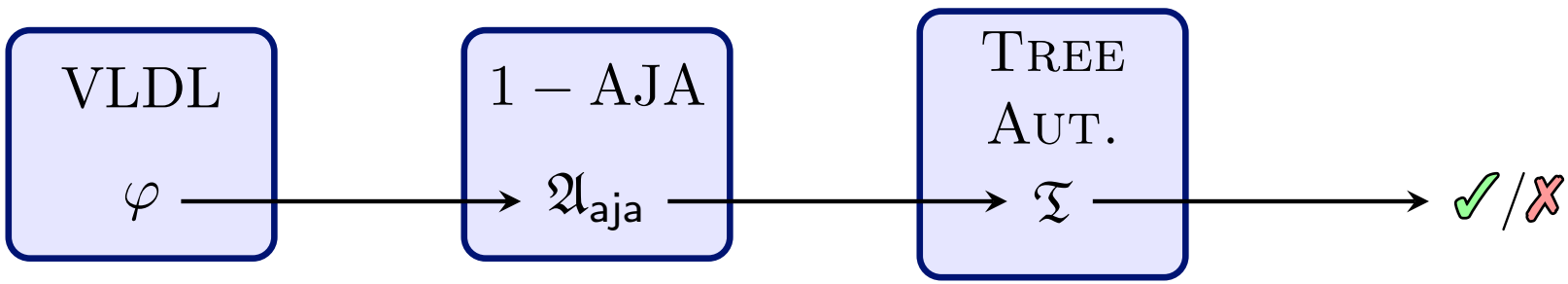

1. What are $1-\mathrm{AJAs}$?
2. How to translate $1-\mathrm{AJAs}$ into tree automata?

Intermediate Automata: 1 – AJA[4]

Extension of alternating automata: $\delta(q, a) = q_1 \wedge (q_2 \vee q_3)$.

$$l \quad c \quad l \quad l \quad r \quad c \quad c \quad l \quad r \quad c \quad \cdots$$

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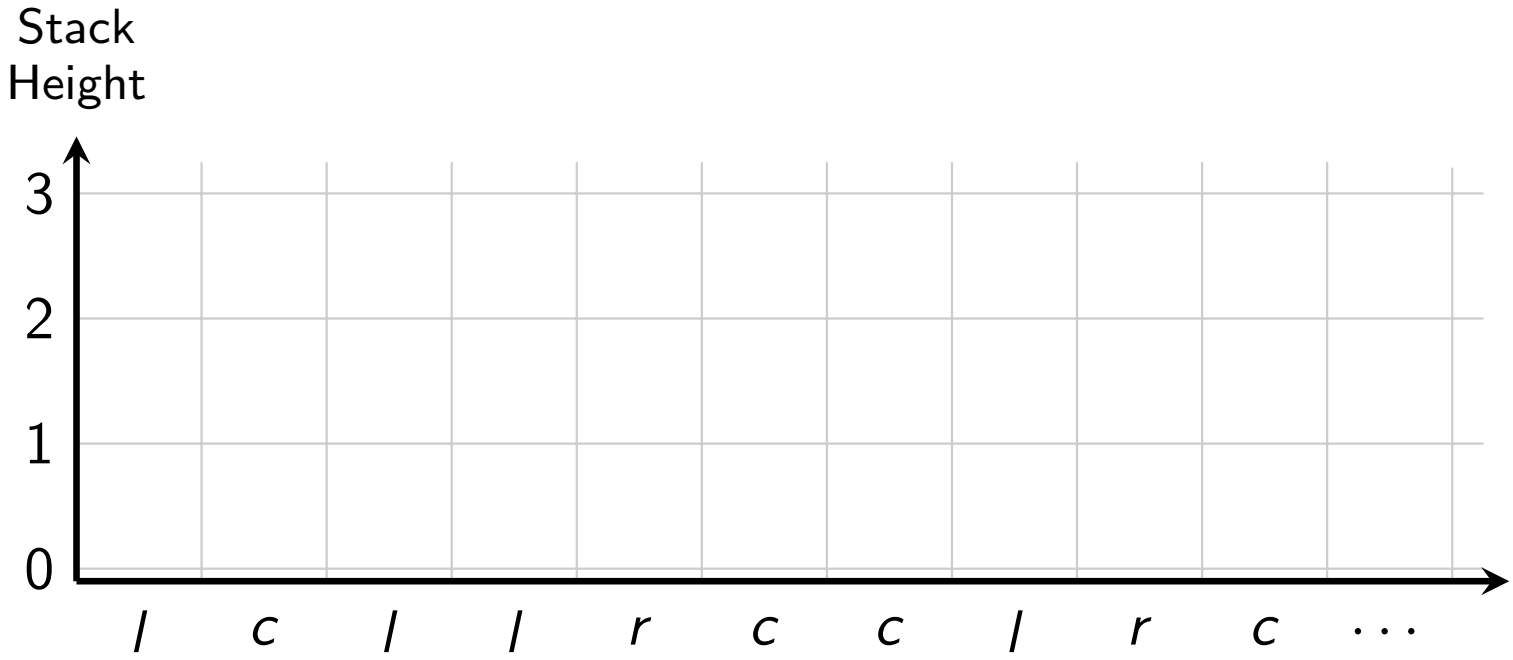

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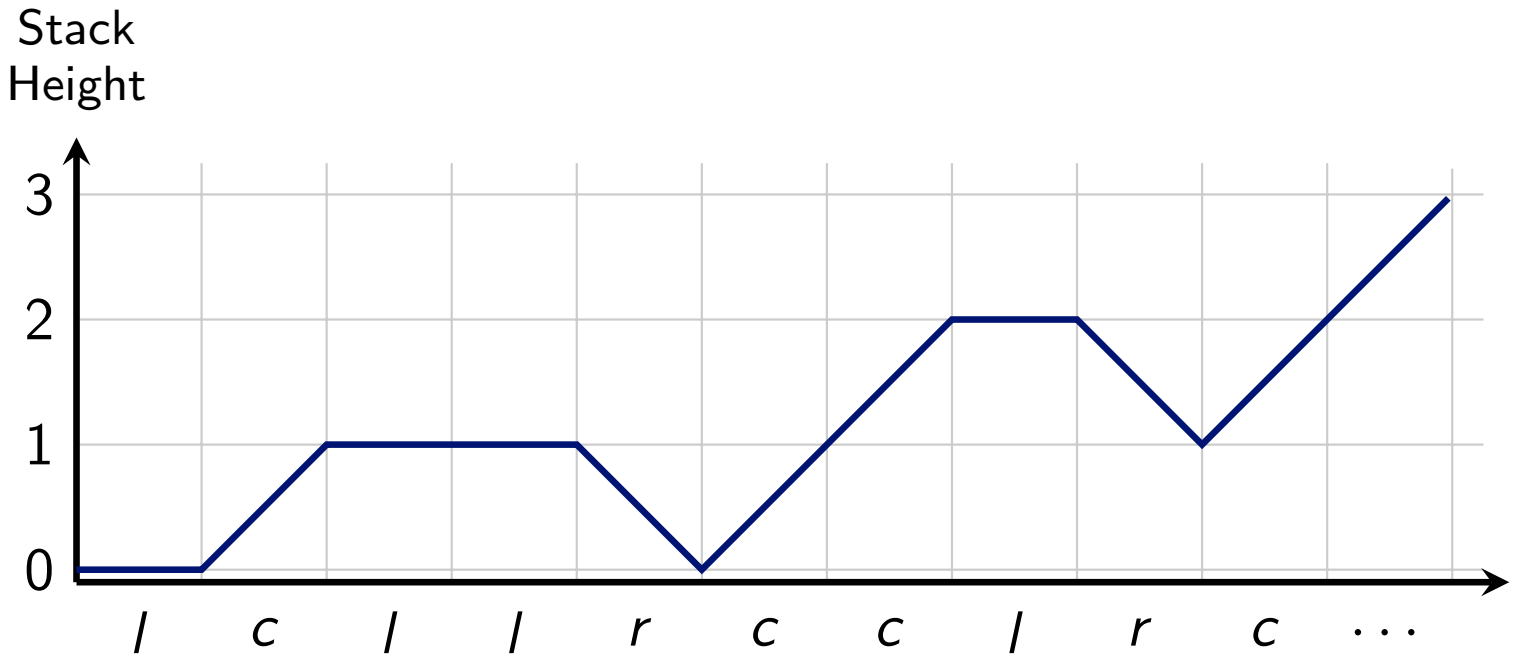

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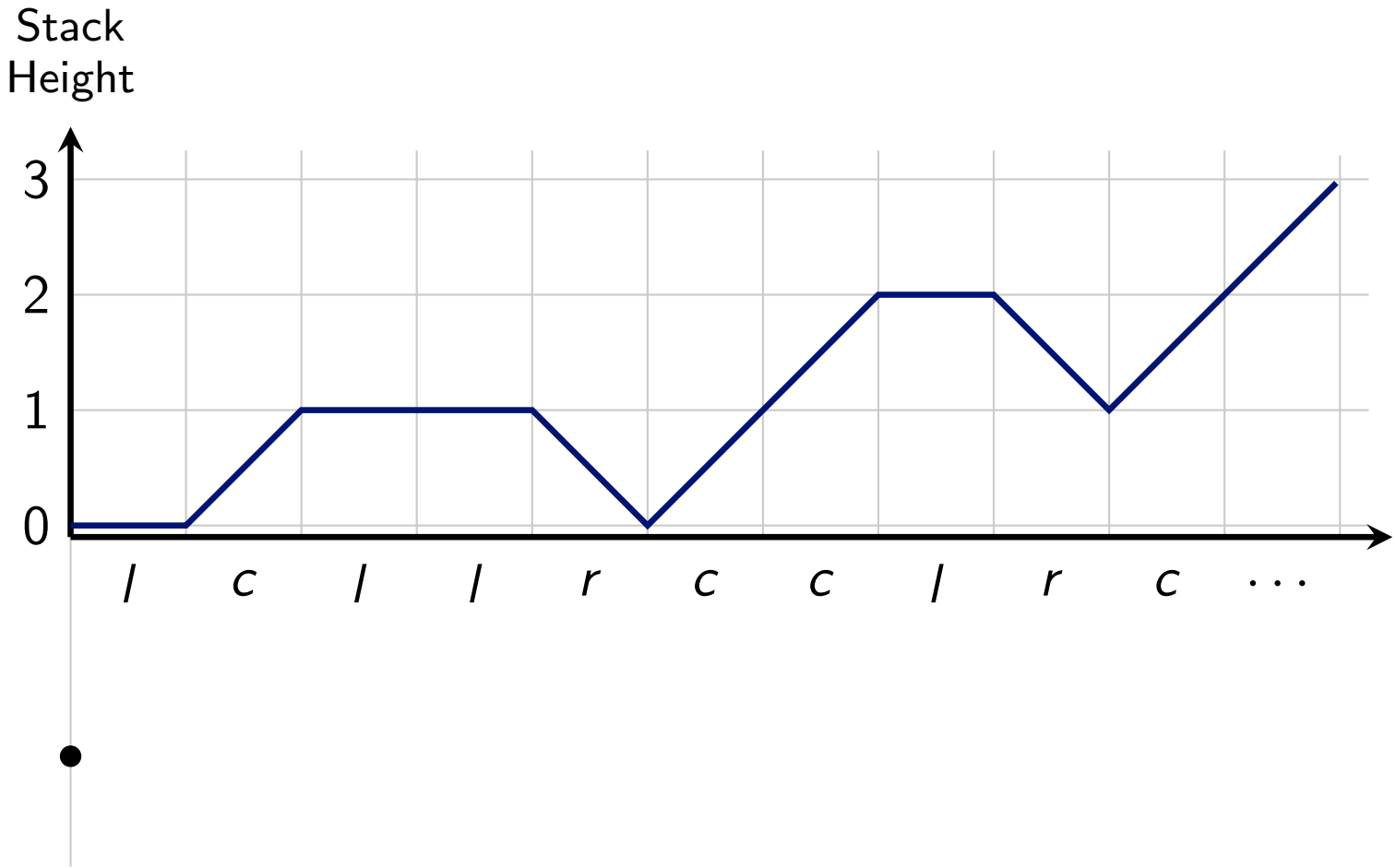

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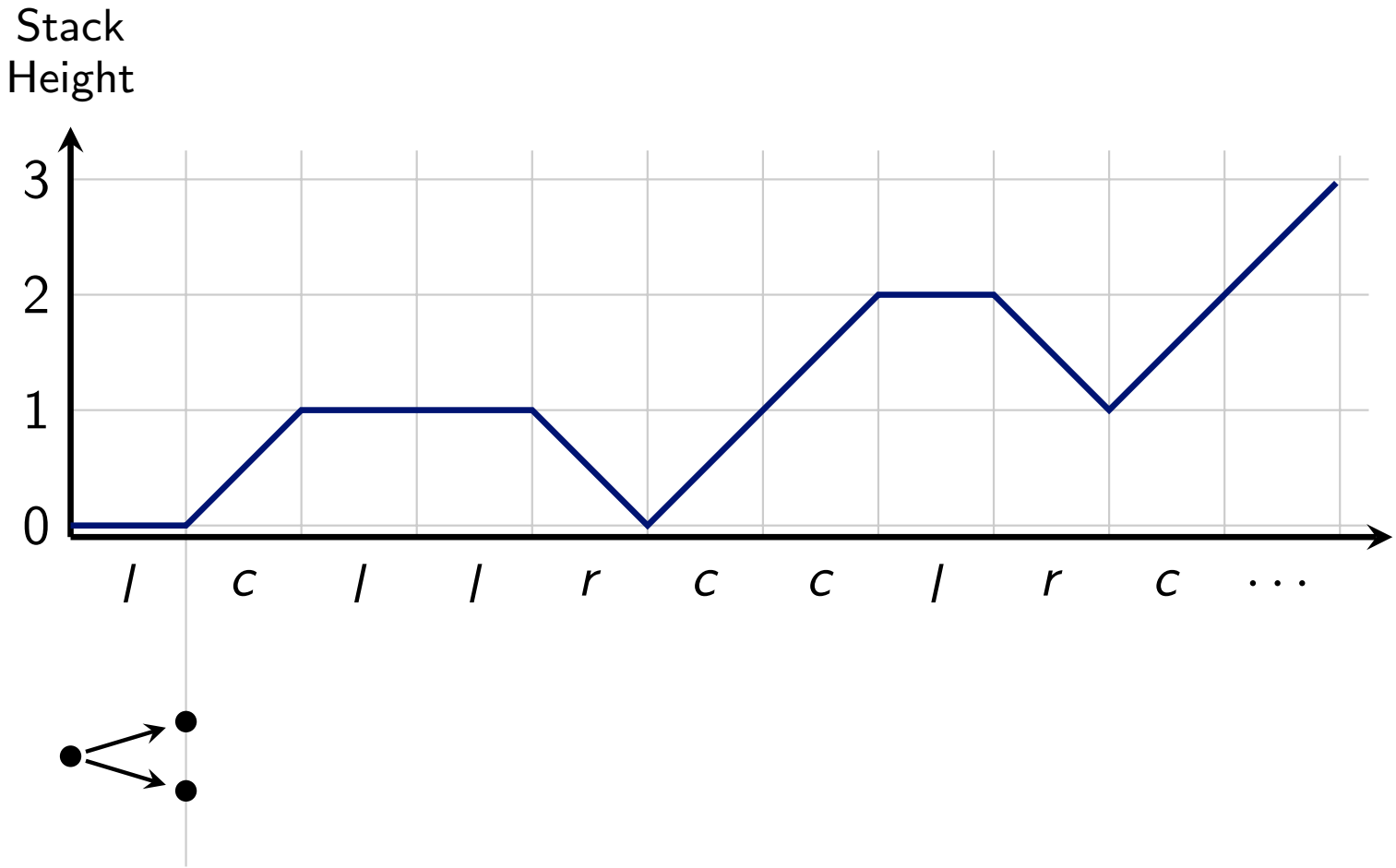

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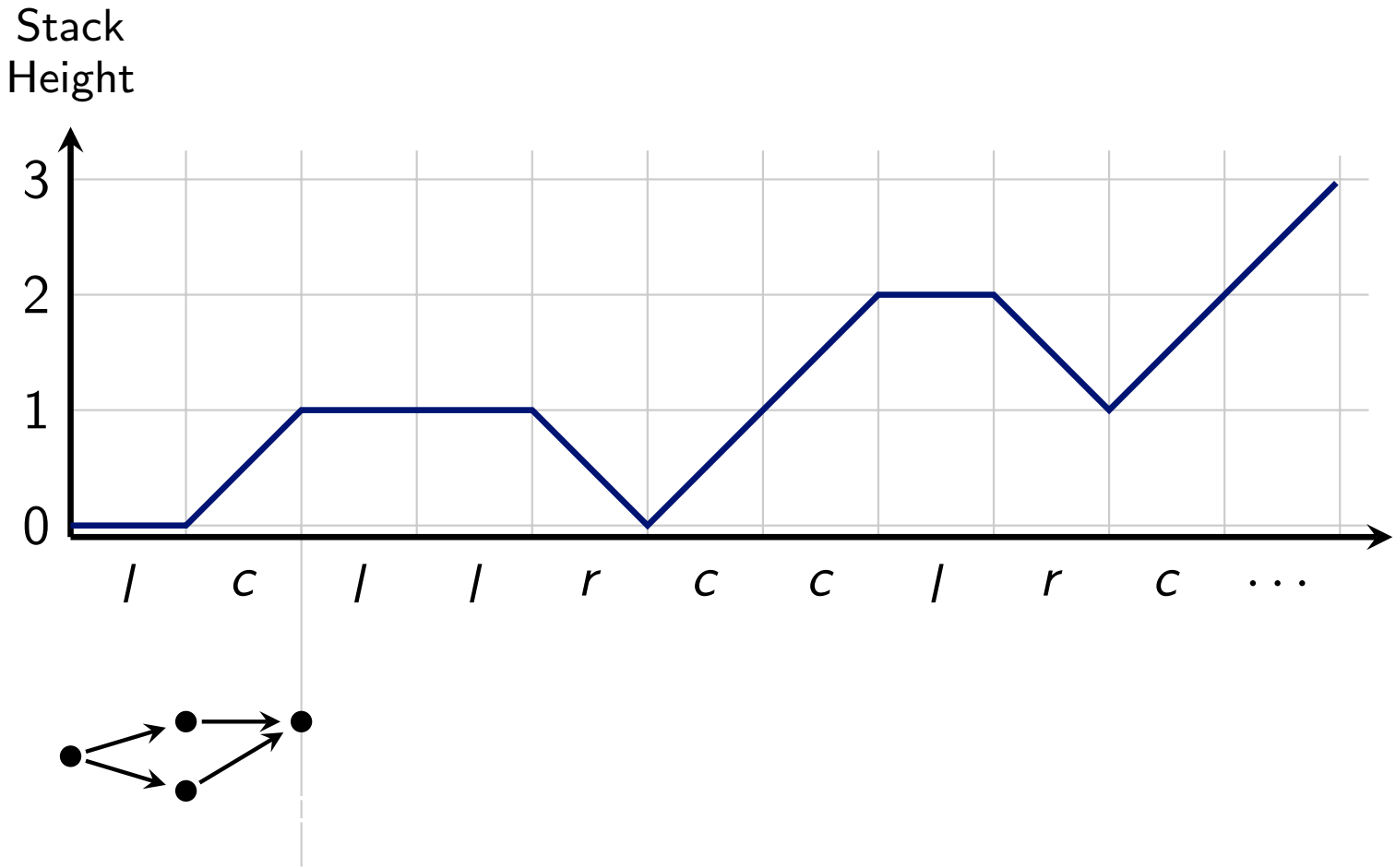

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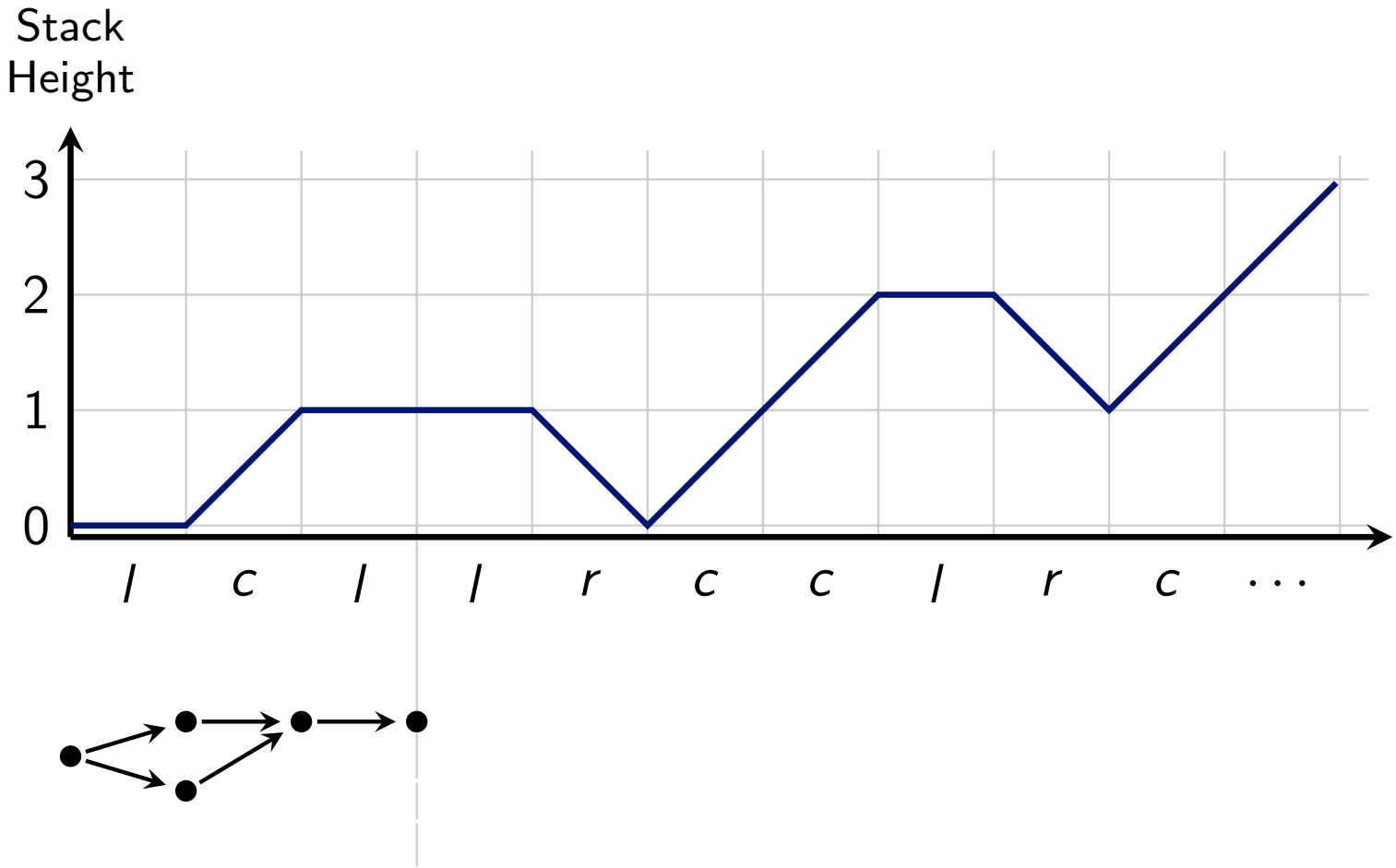

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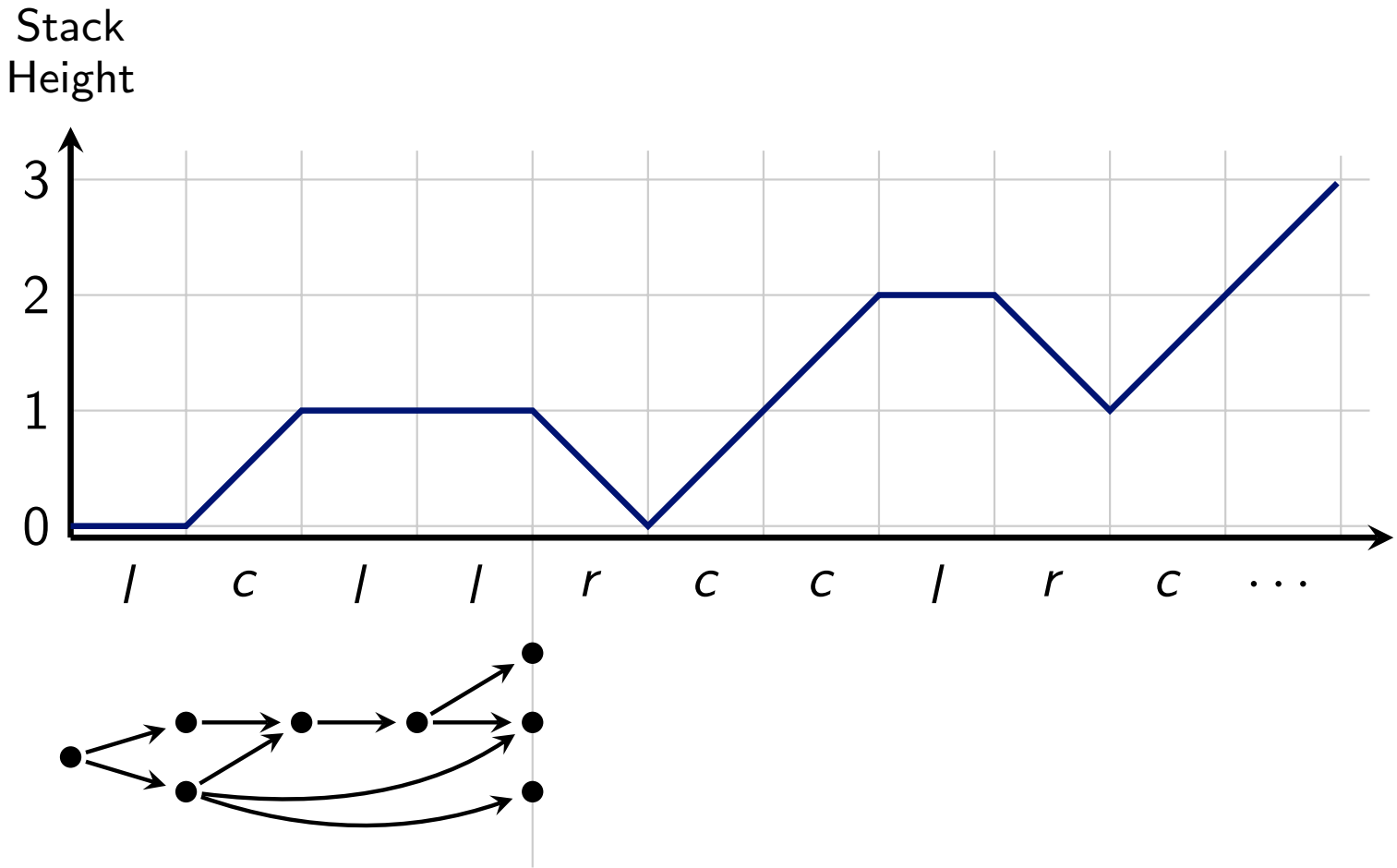

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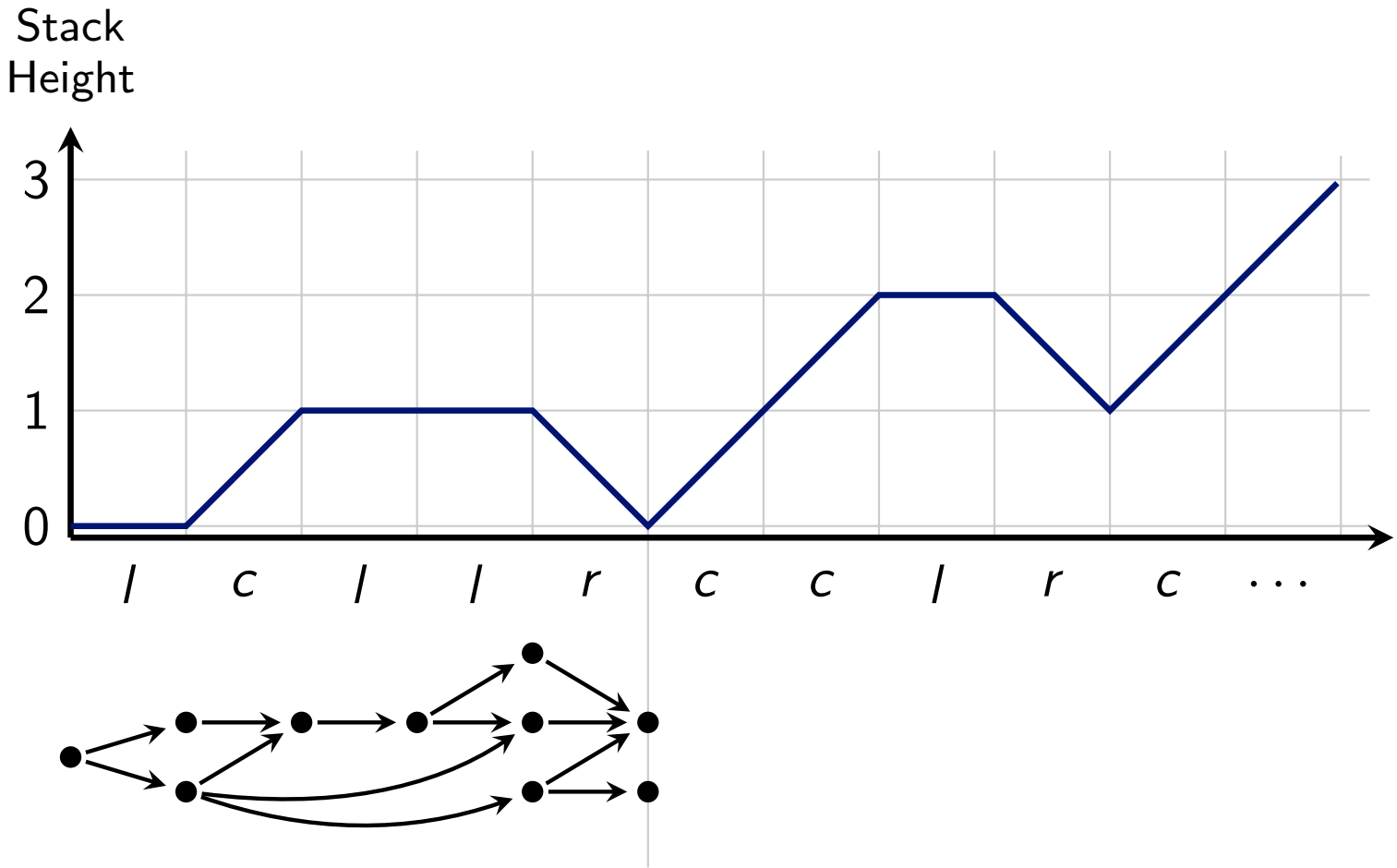

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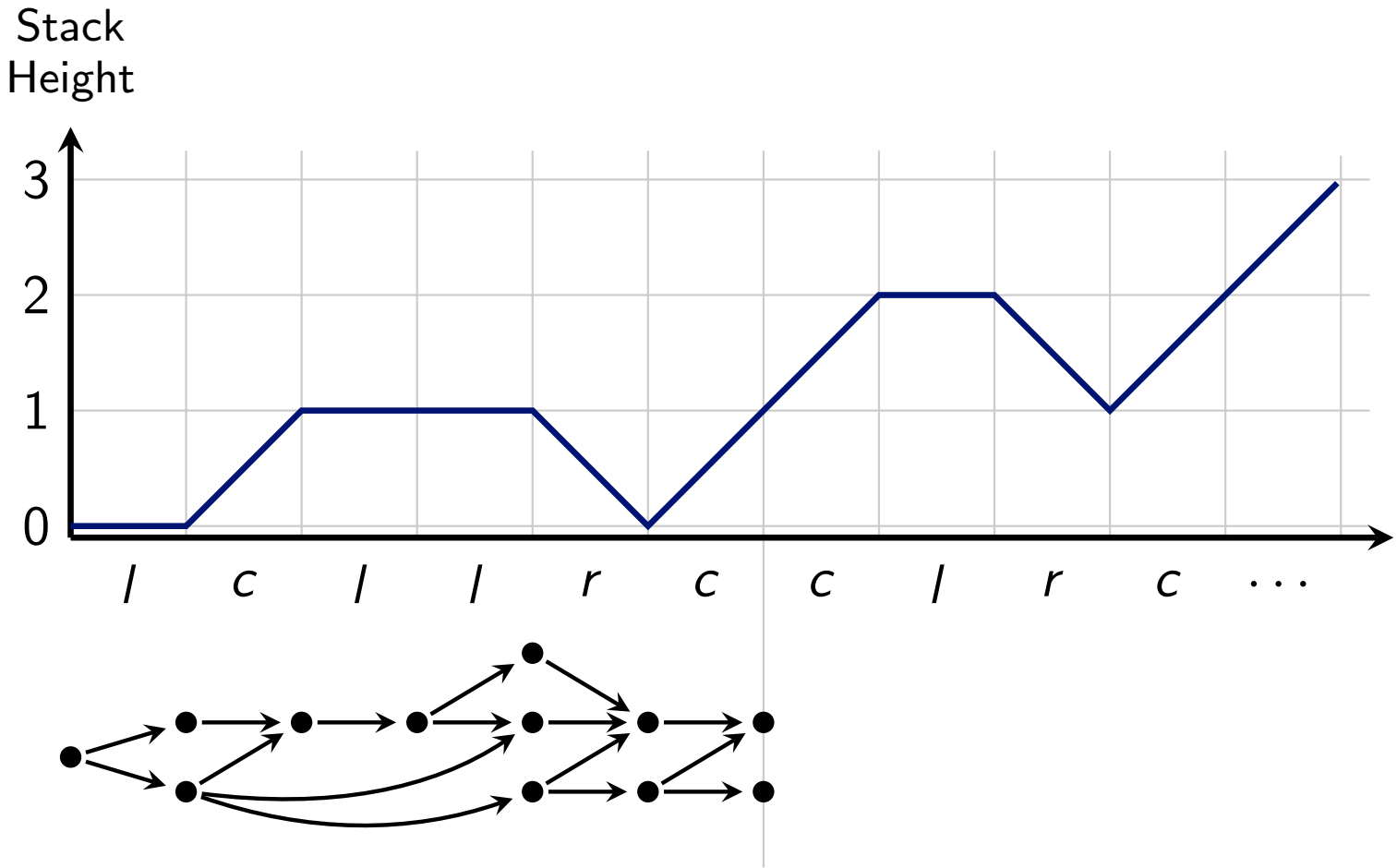

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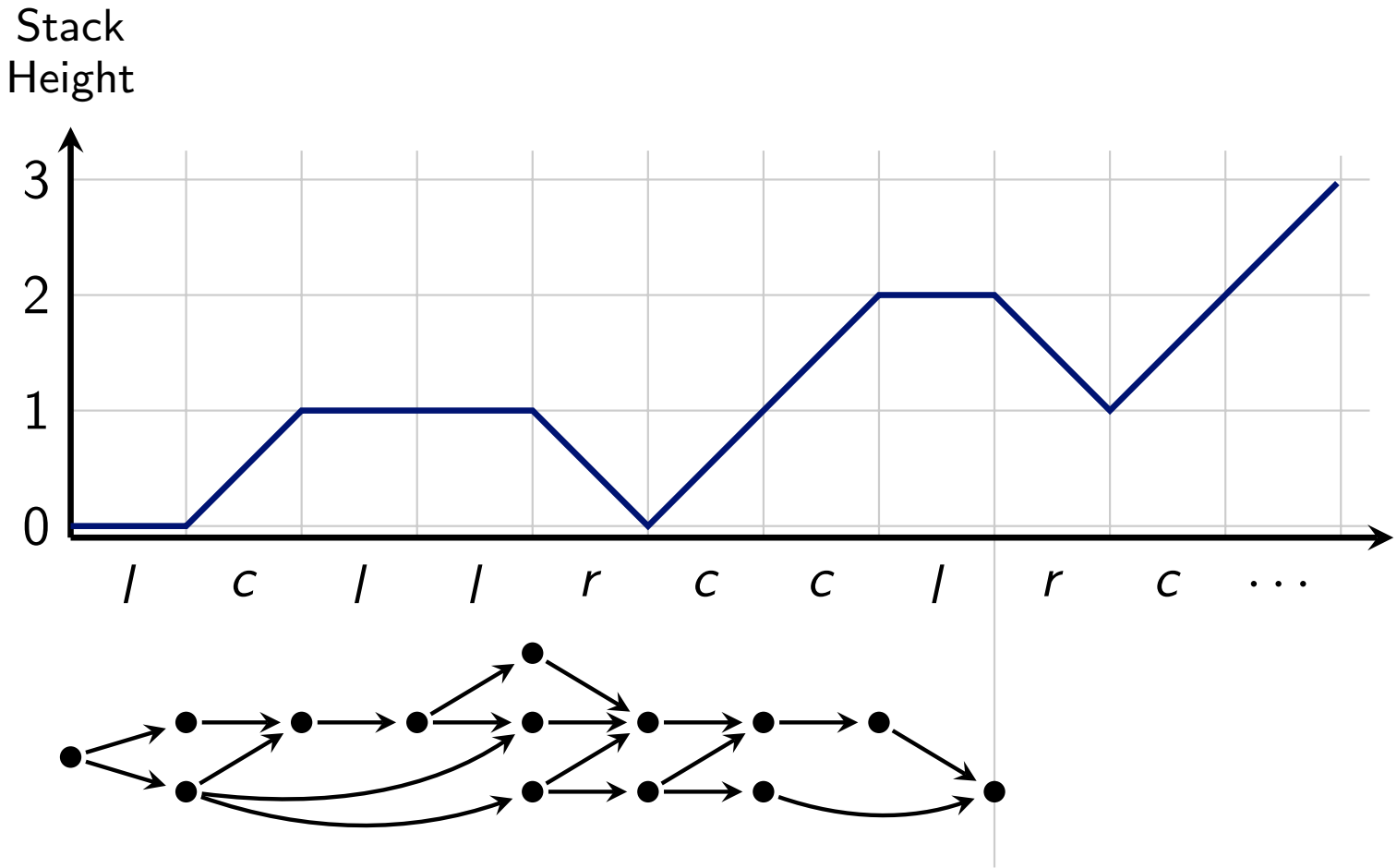
Extension of alternating automata: $\delta(q, a) = q_1 \wedge (q_2 \vee q_3)$.



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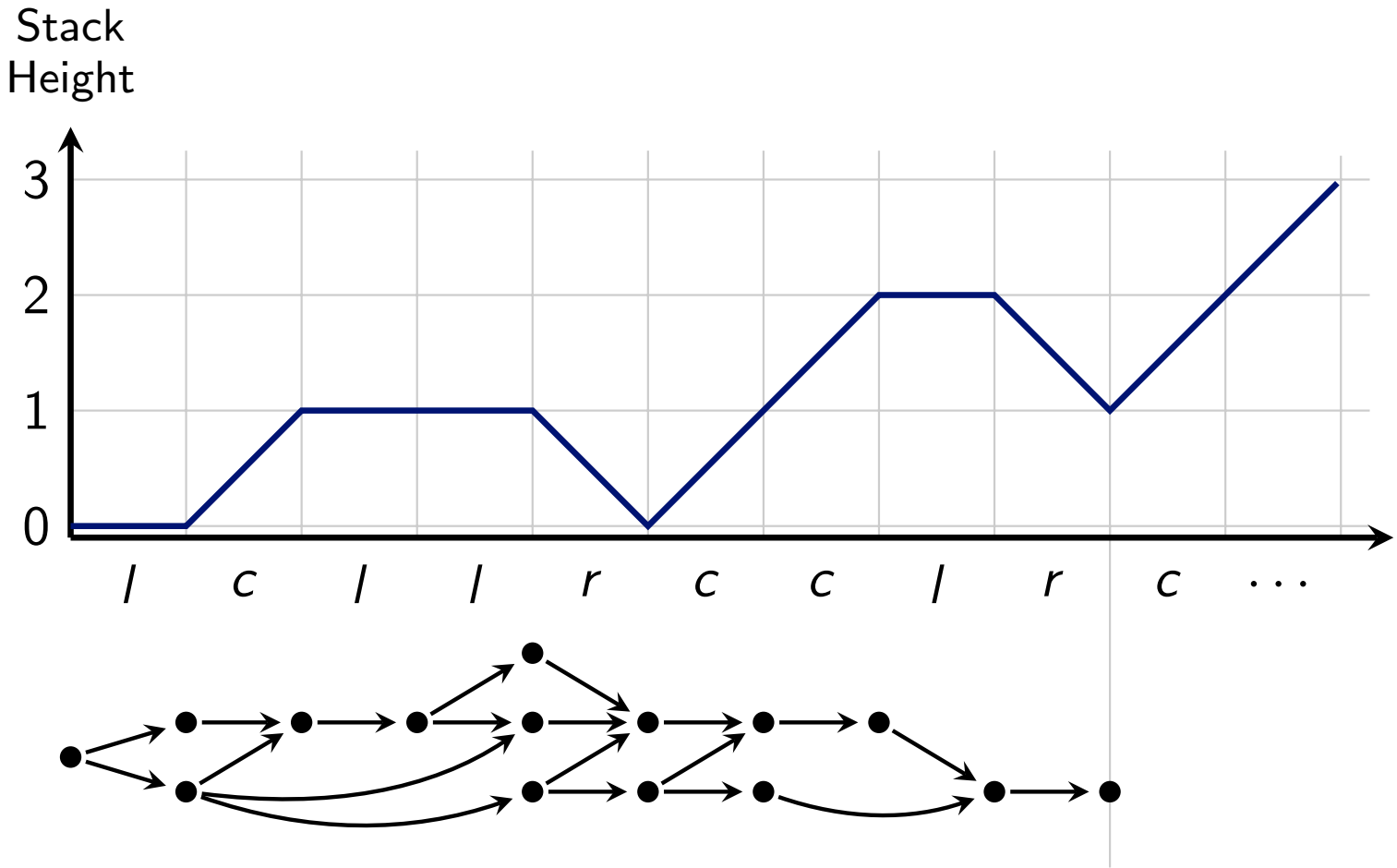

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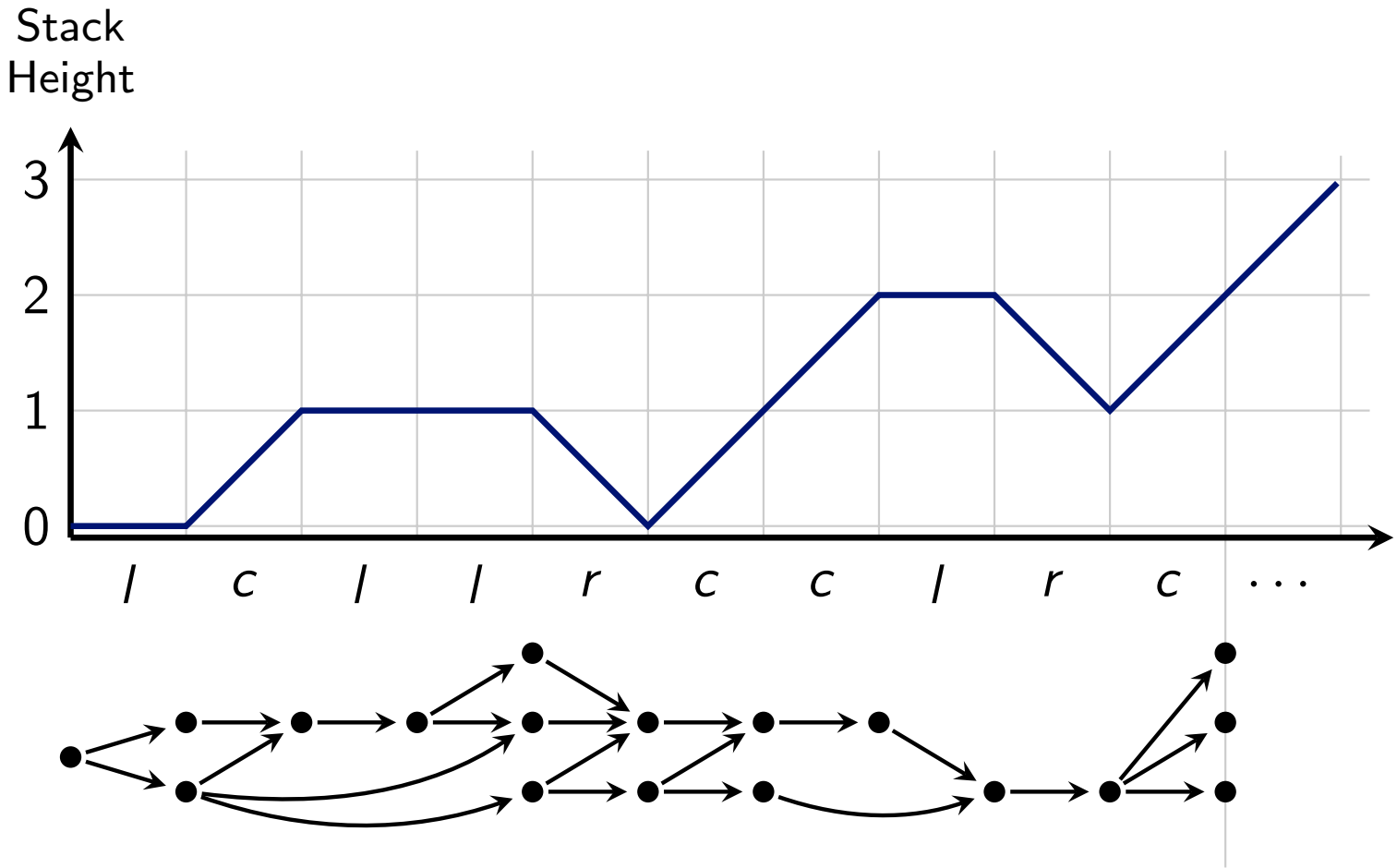

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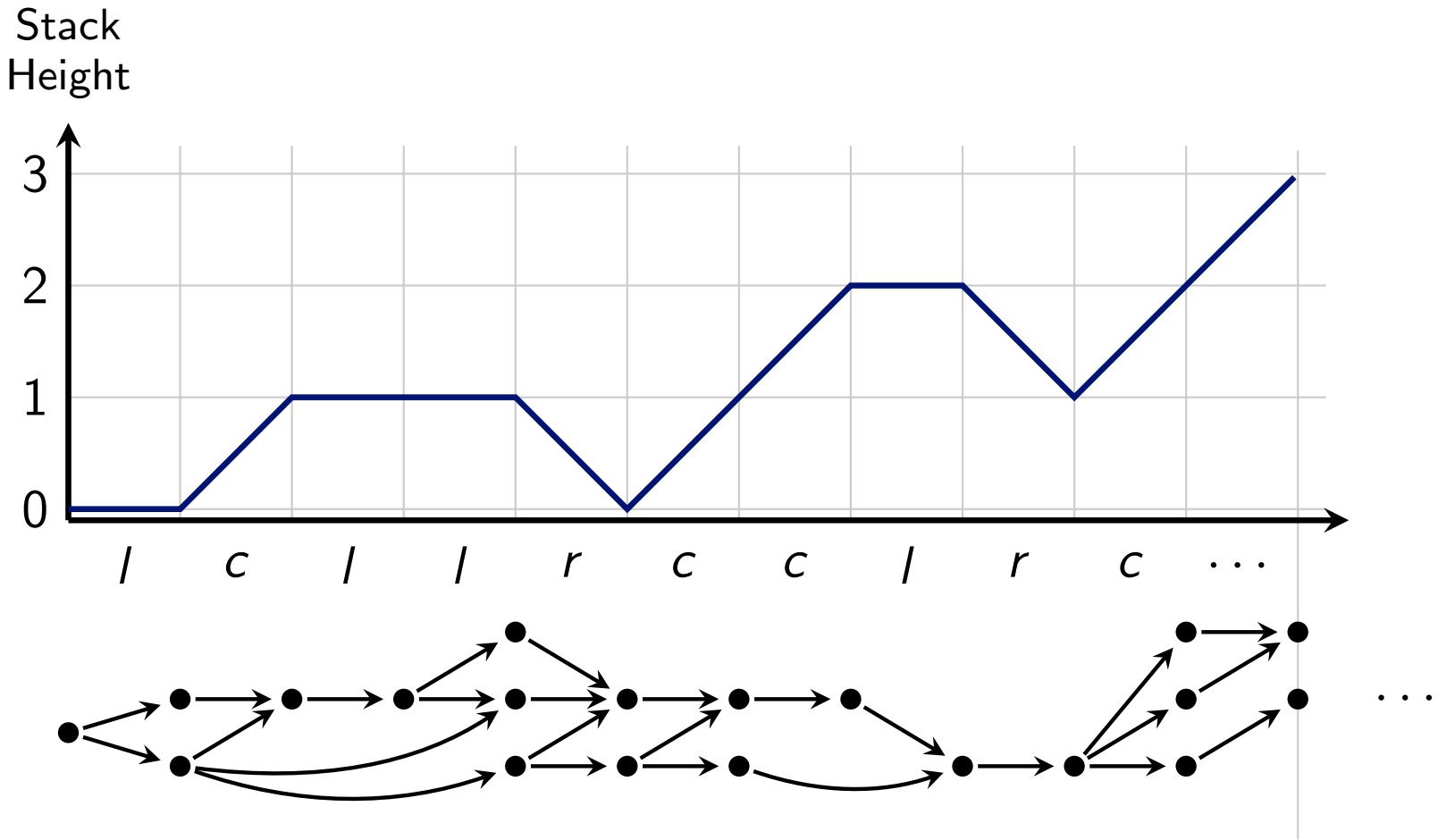

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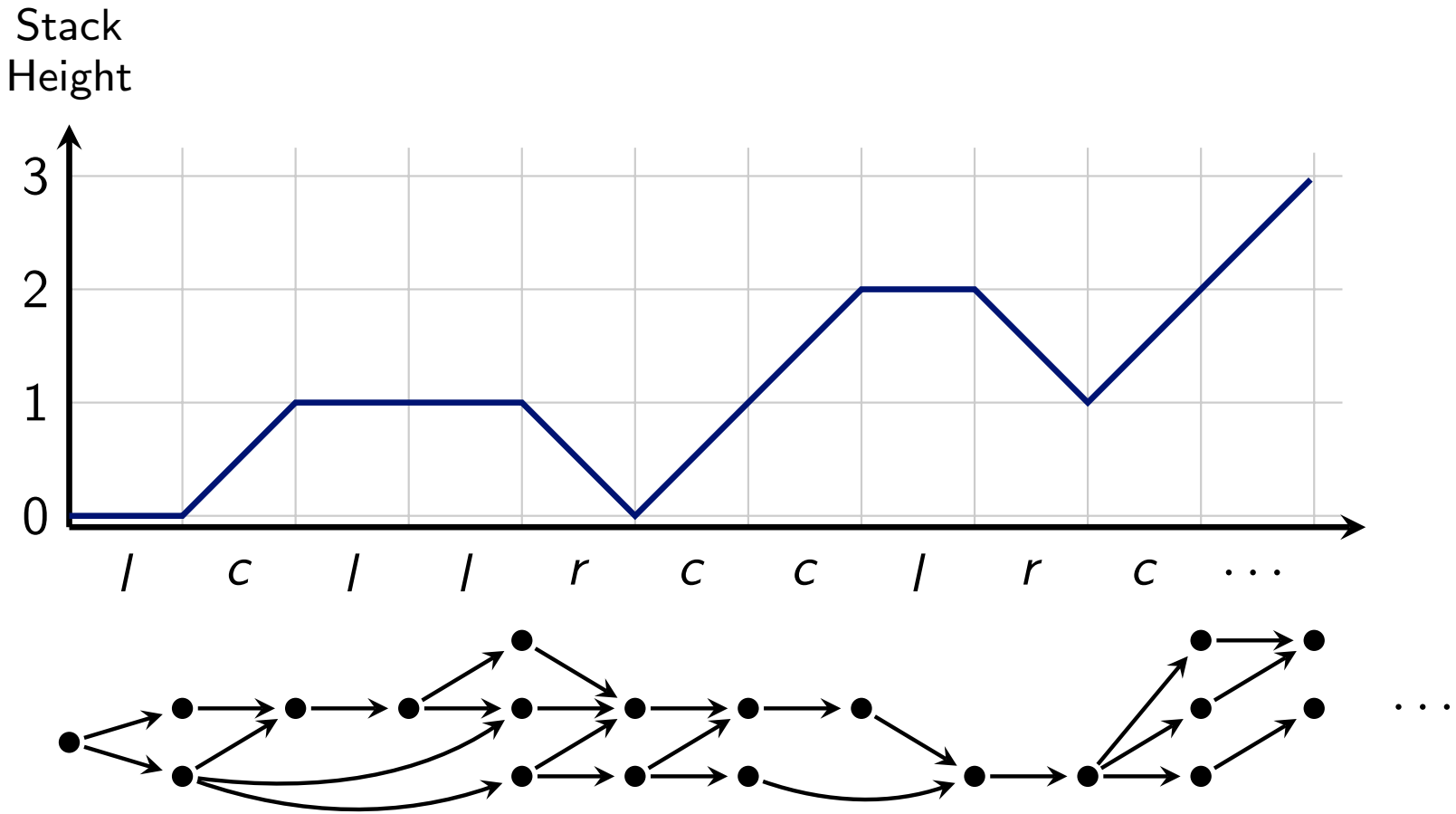

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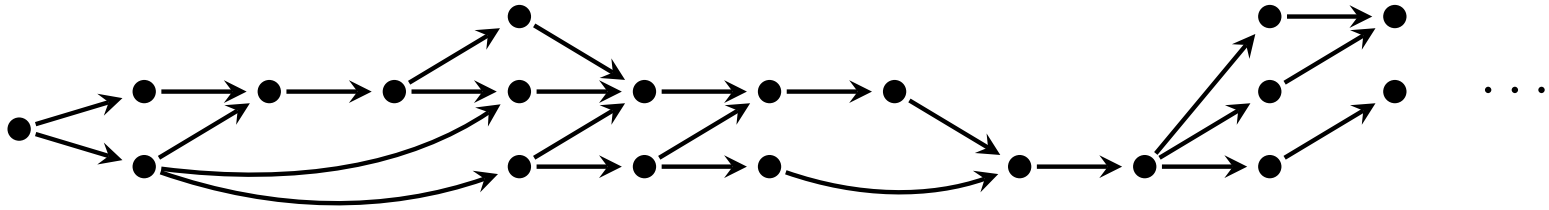

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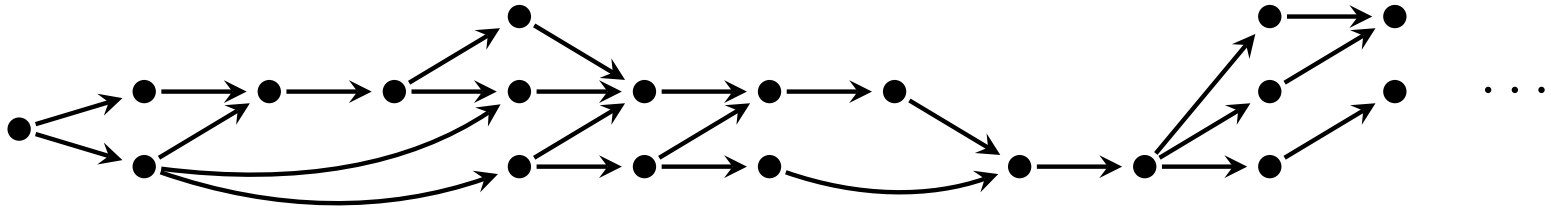
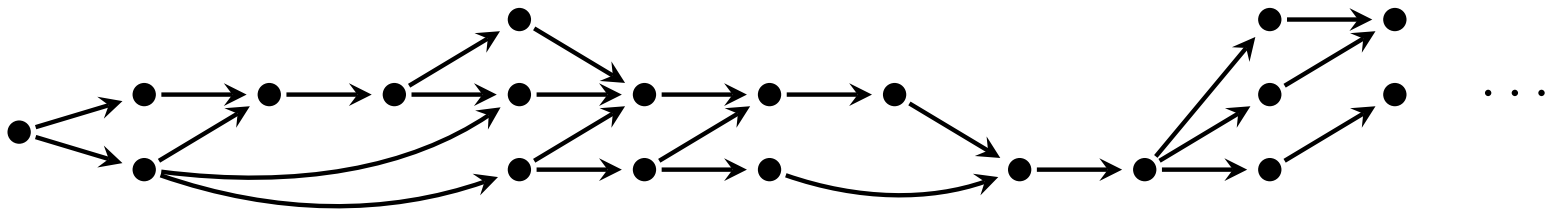


Acceptance: Each branch visits accepting states infinitely often.

Intermediate Automata: $1-\mathrm{AJA}$



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Theorem (Ext. of (W. and Zimmermann, 2016))

For each VLDL *formula there exists an equivalent* $1-\mathrm{AJA}$ *of polynomial size.*

Guiding Questions

1. What are $1-\mathrm{AJAs}$?
2. How to translate $1-\mathrm{AJAs}$ into tree automata?

Guiding Questions

1. What are $1-\mathrm{AJAs}$? ✓
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Guiding Questions

1. What are $1-\mathrm{AJAs}$? ✓
2. How to translate $1-\mathrm{AJAs}$ into tree automata?
   - How to translate words into trees?

Encoding Words as Trees

$$/ \quad c \quad / \quad / \quad r \quad c \quad c \quad / \quad r \quad c \quad c \quad \cdots$$

Adapted from (Alur and Madhusudan, 2004)

Encoding Words as Trees

$l \quad c \quad l \quad l \quad r \quad c \quad c \quad l \quad r \quad c \quad c \quad \cdots$

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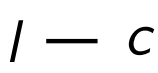

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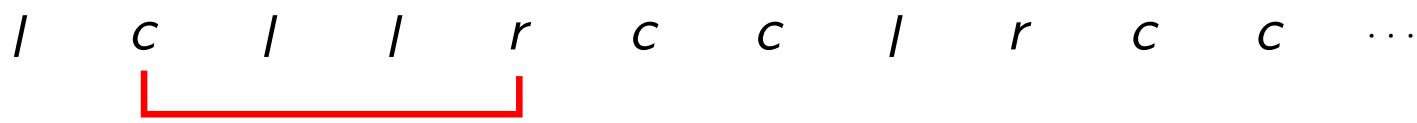

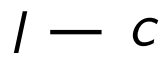

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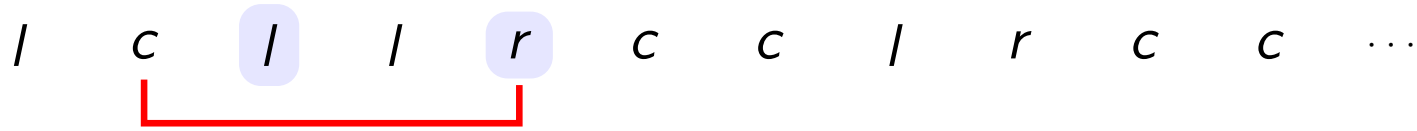

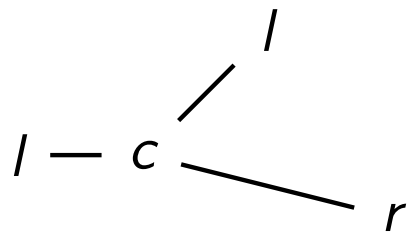

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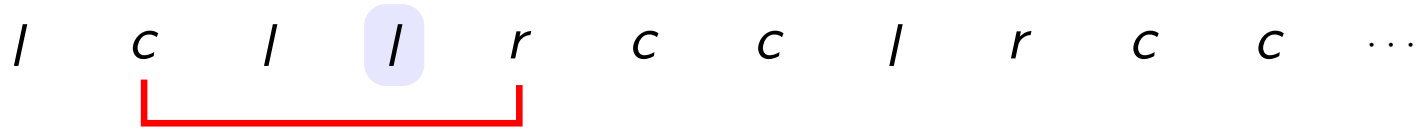

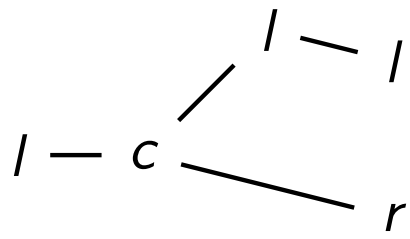

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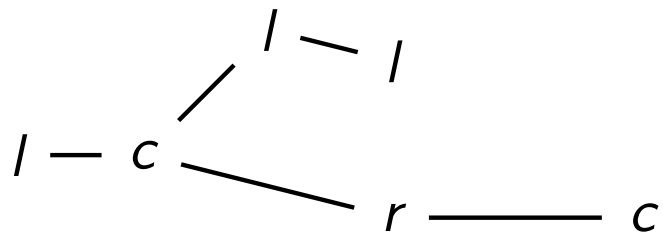

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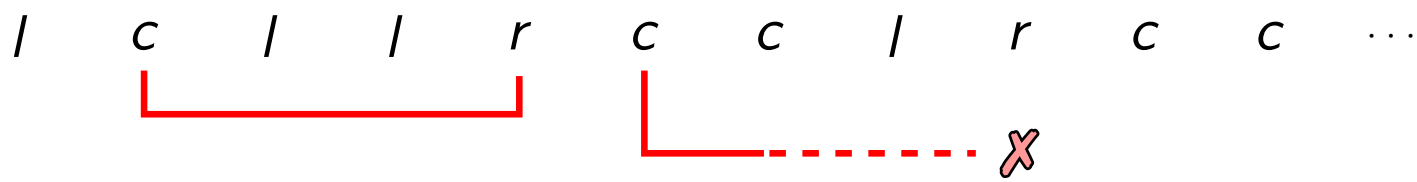

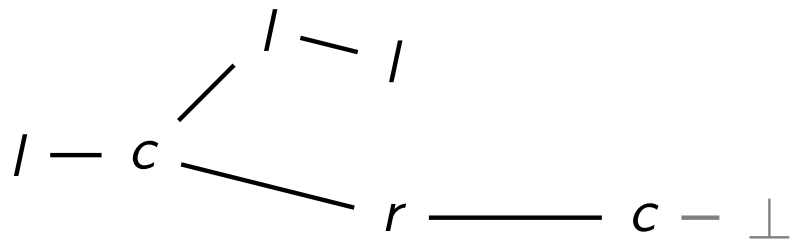

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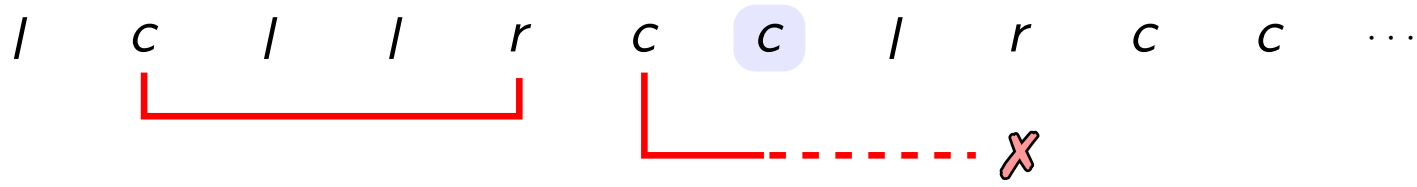

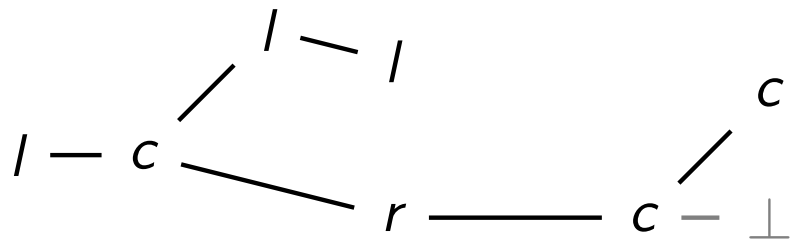

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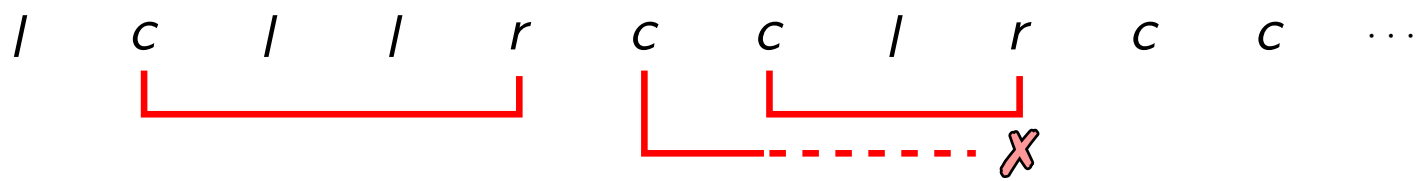

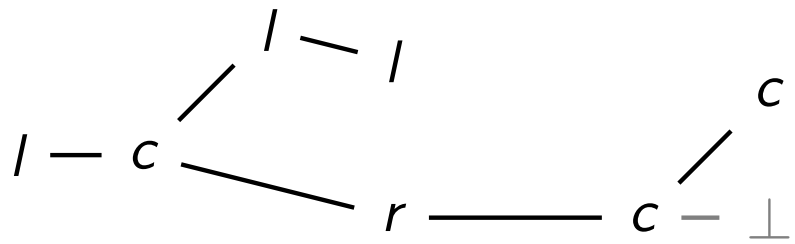

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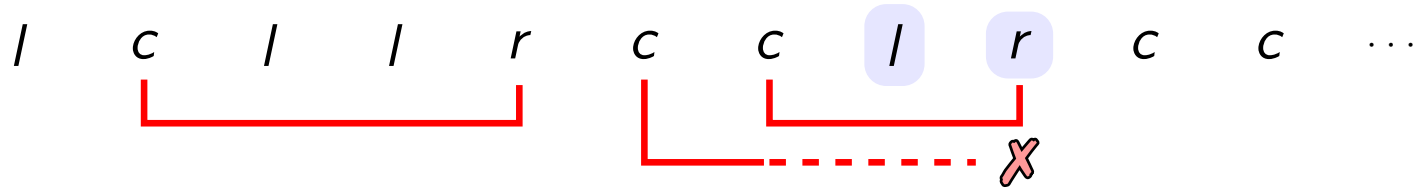

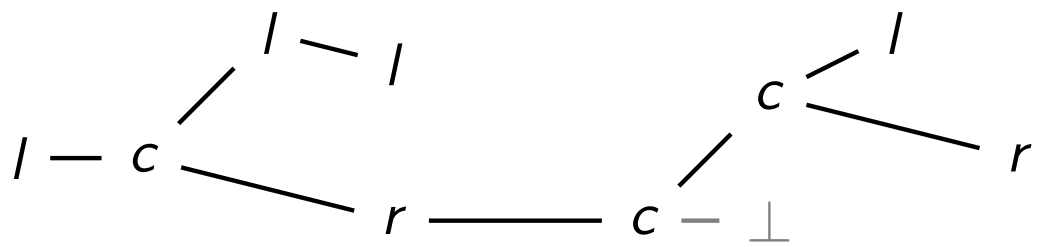

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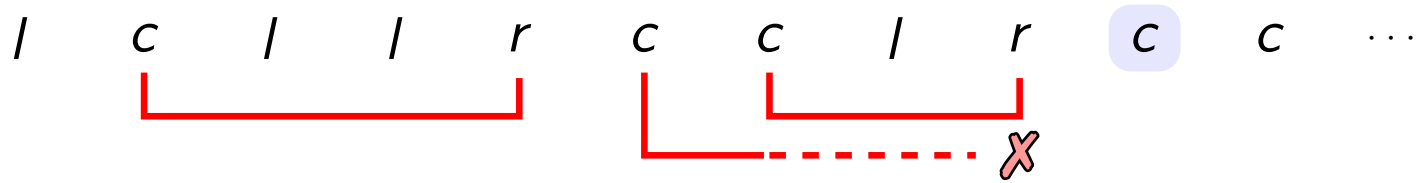

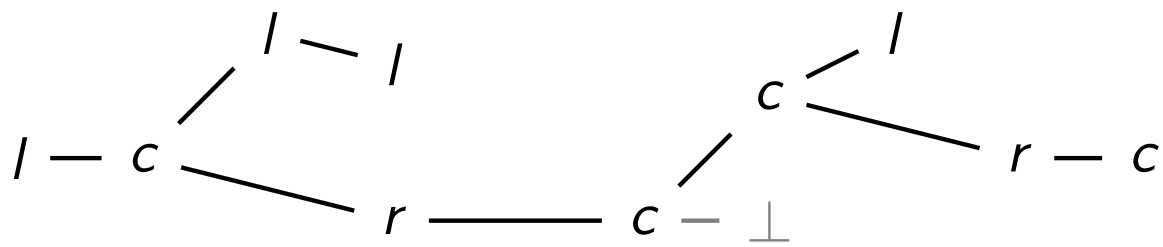

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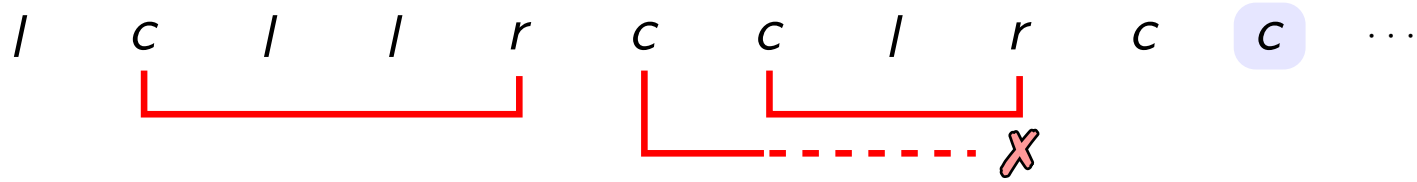

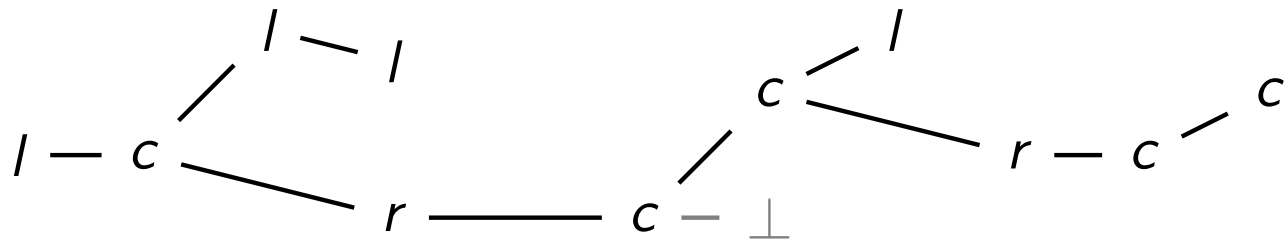

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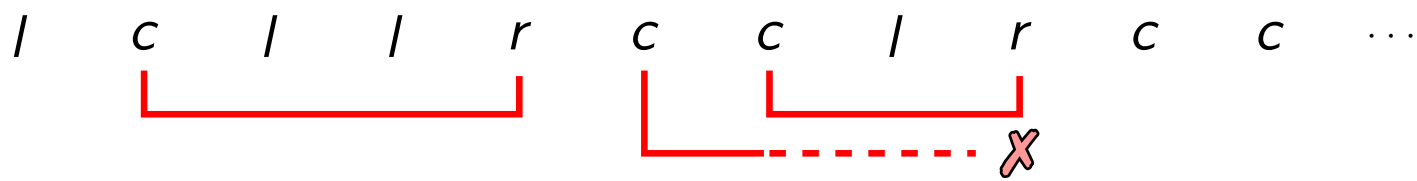

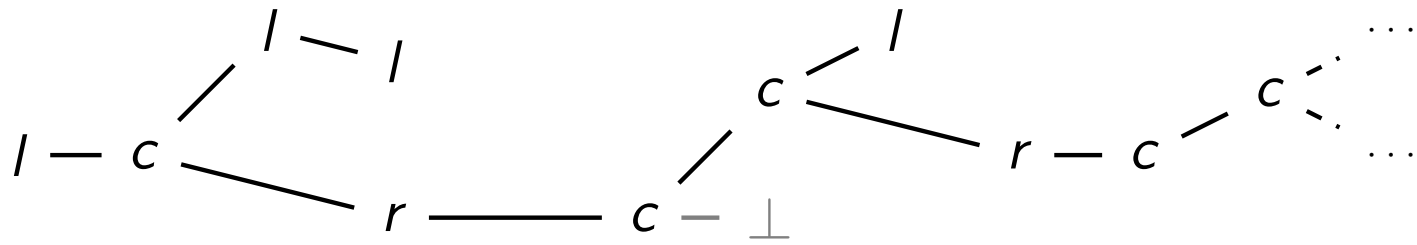

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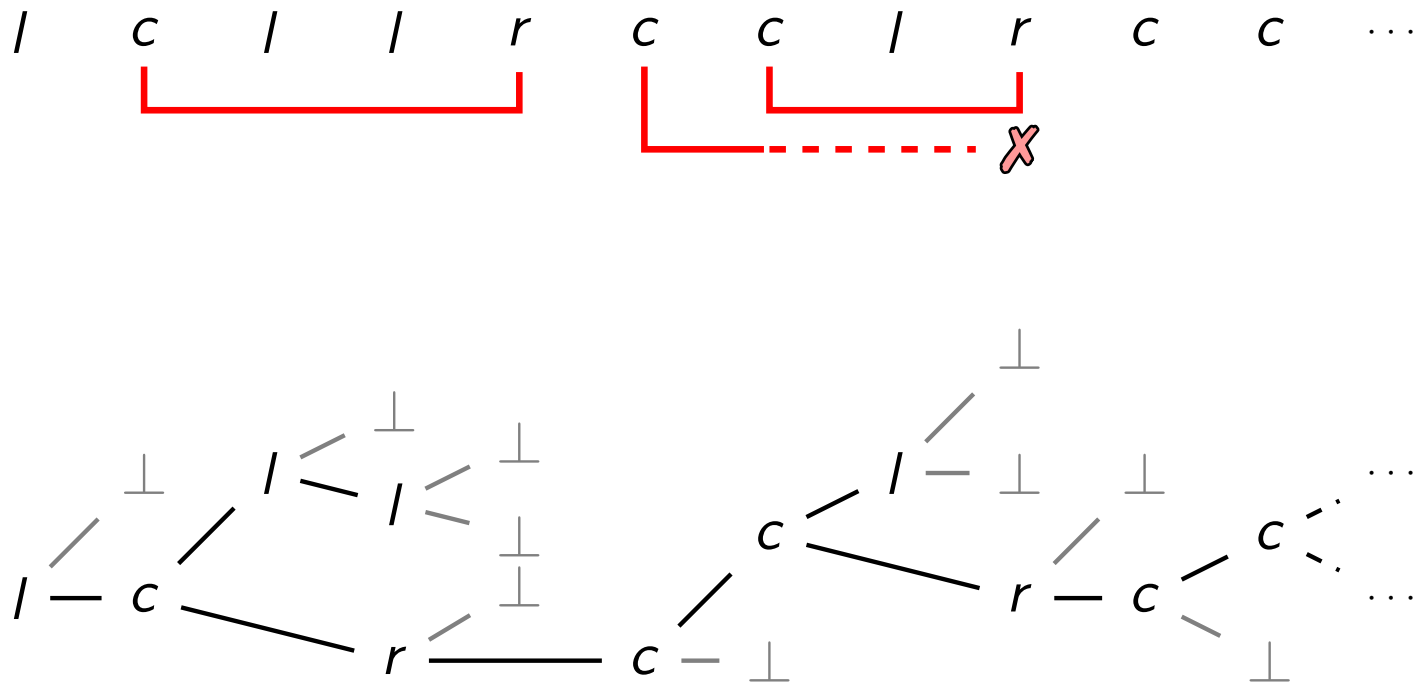

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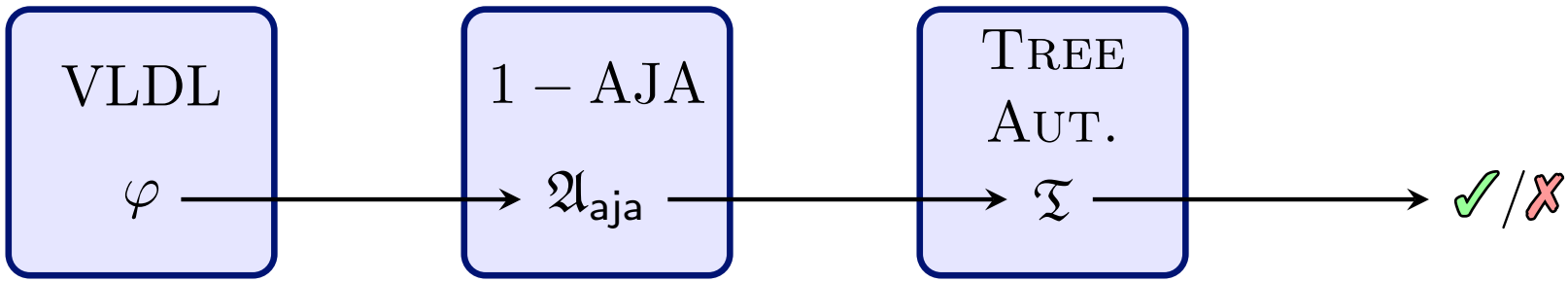
Overview


VLDL
φ
$1 - \mathrm{AJA}$
$\mathfrak{A}_{\mathsf{aja}}$
Tree Aut.
$\mathfrak{T}$
✓/✗

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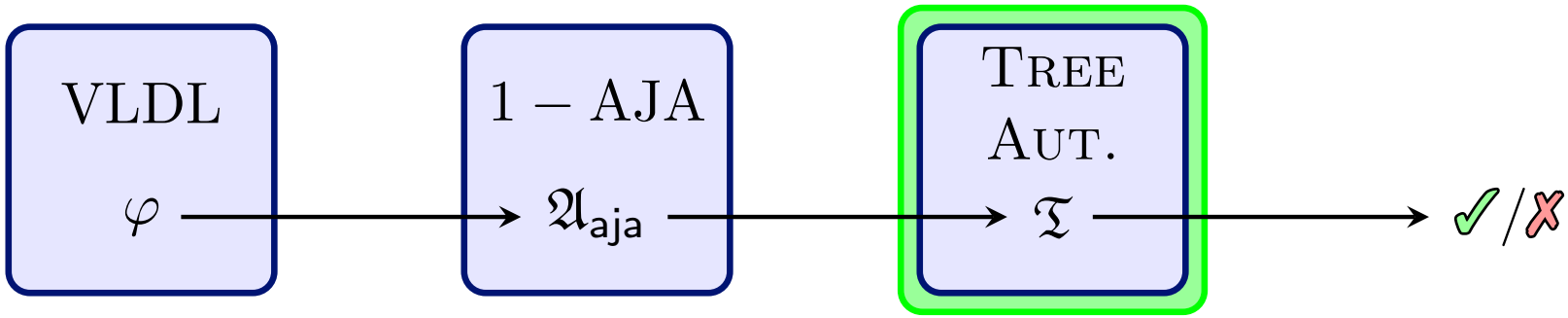
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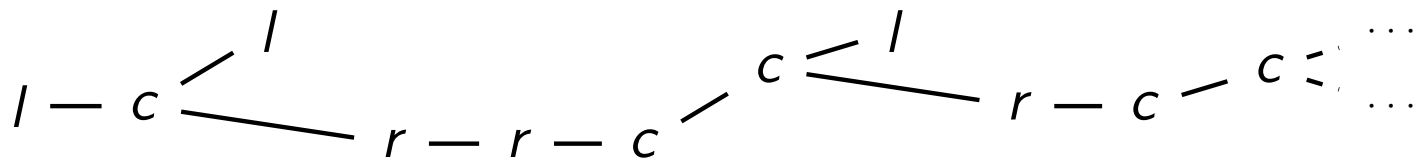
Tree Automata



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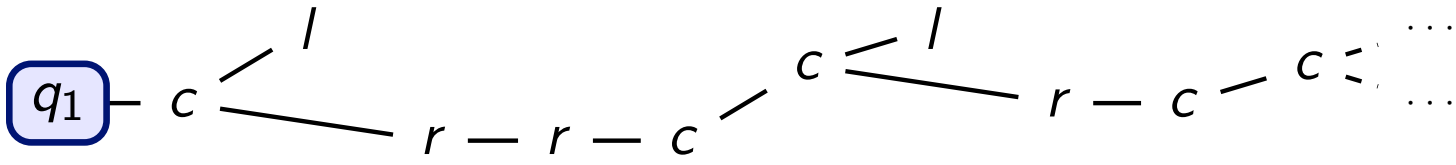

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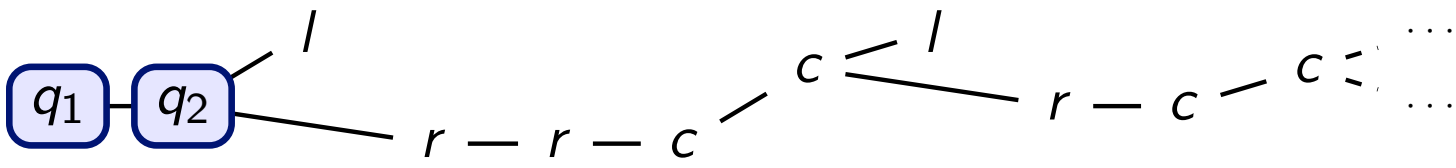

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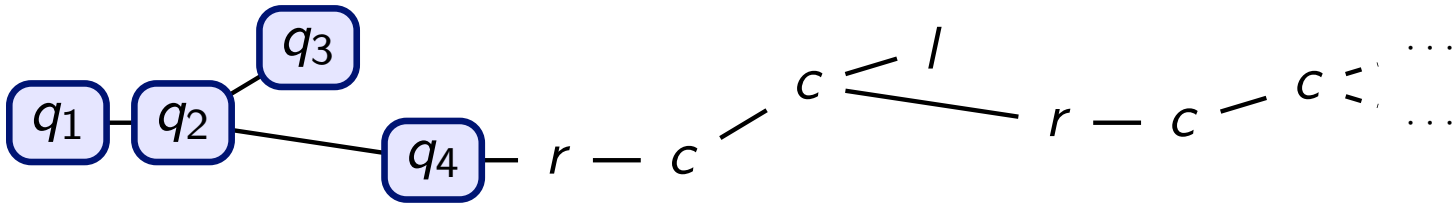

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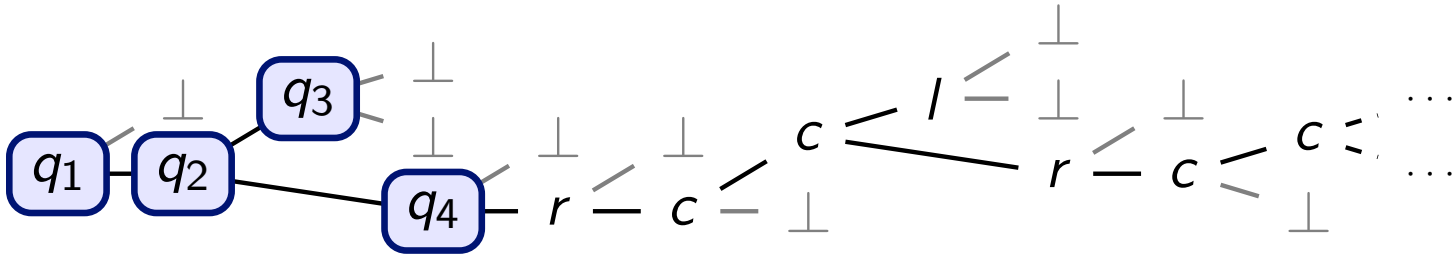

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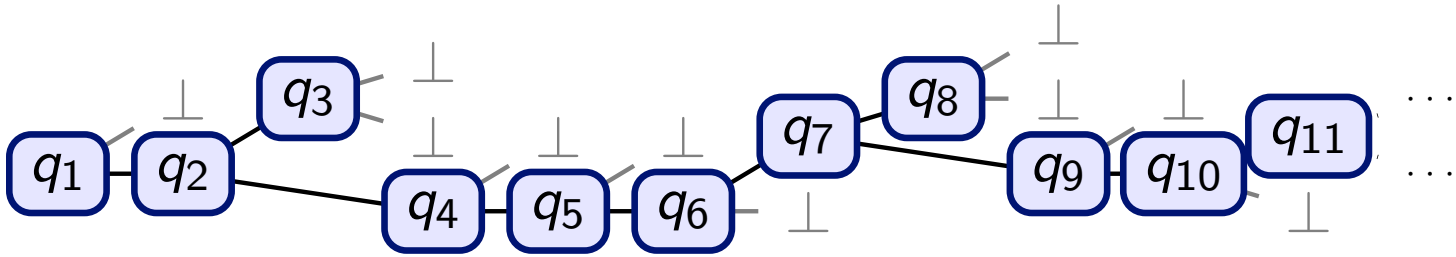

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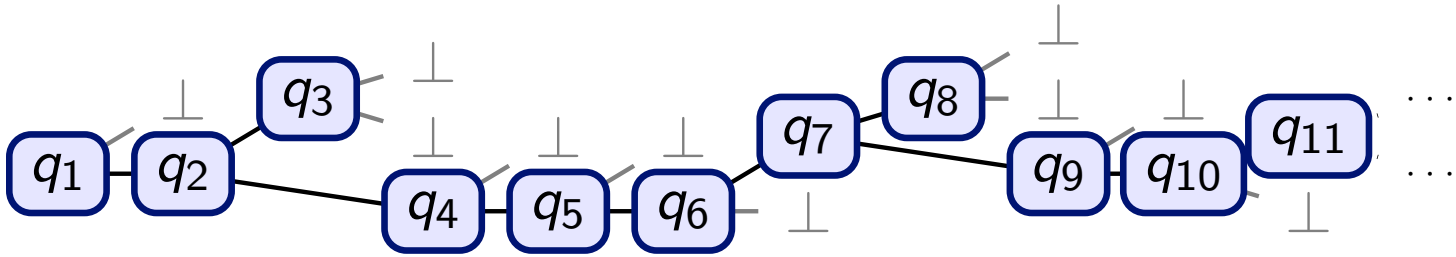
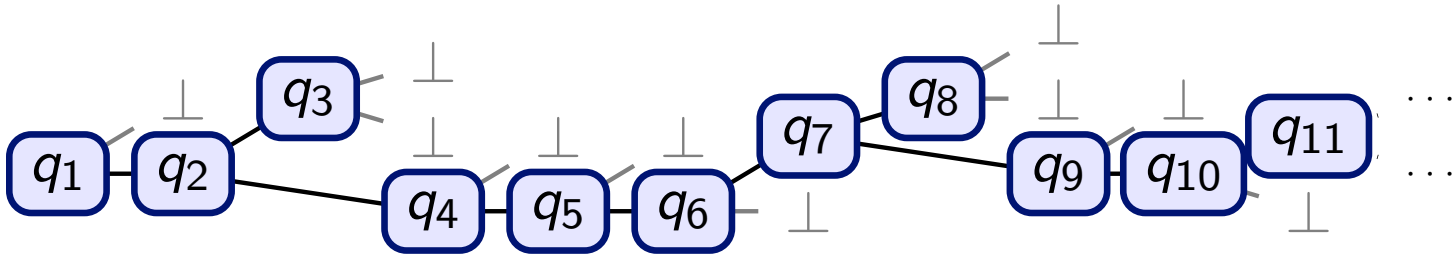


Acceptance: Every branch has infinitely many accepting vertices

Tree Automata



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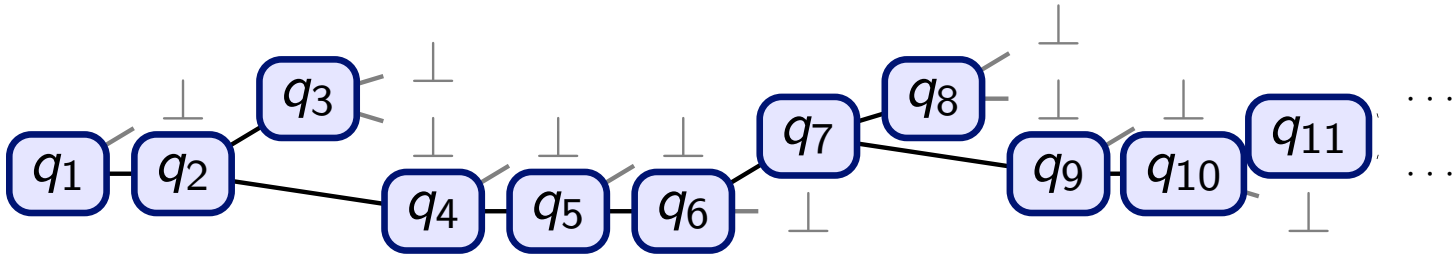
Component	Technique

Tree Automata



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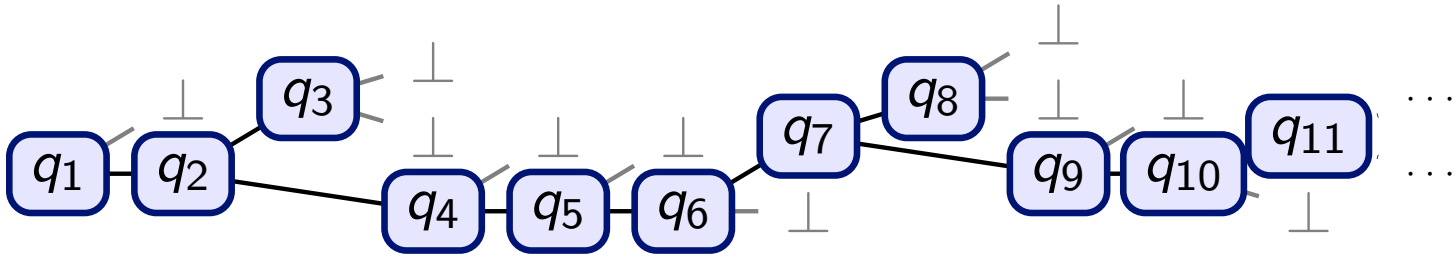
Component	Technique
States	

Tree Automata



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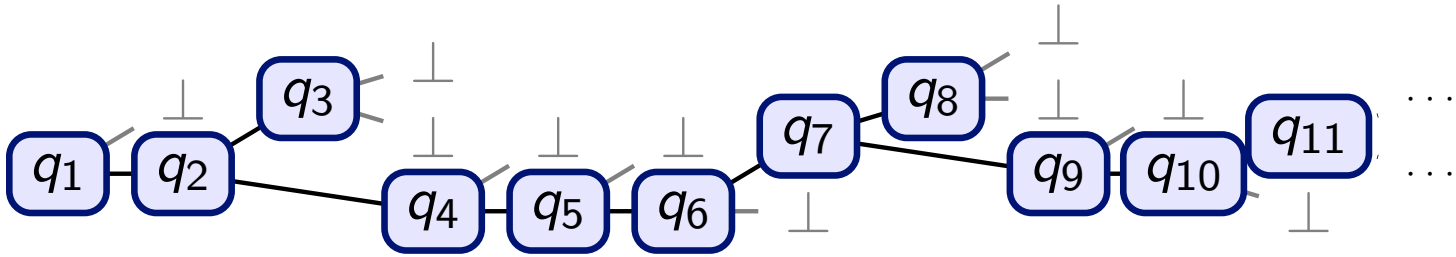
Component	Technique
States	
Transitions	

Tree Automata



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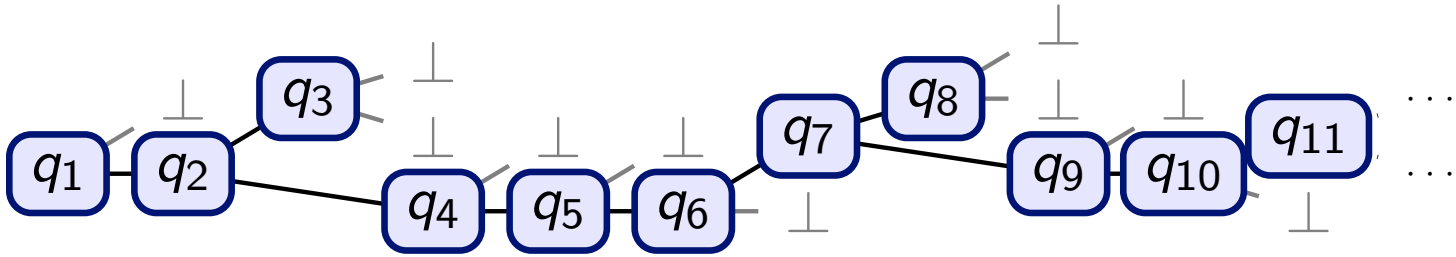
Component	Technique
States	
Transitions	
Accepting States	

Tree Automata



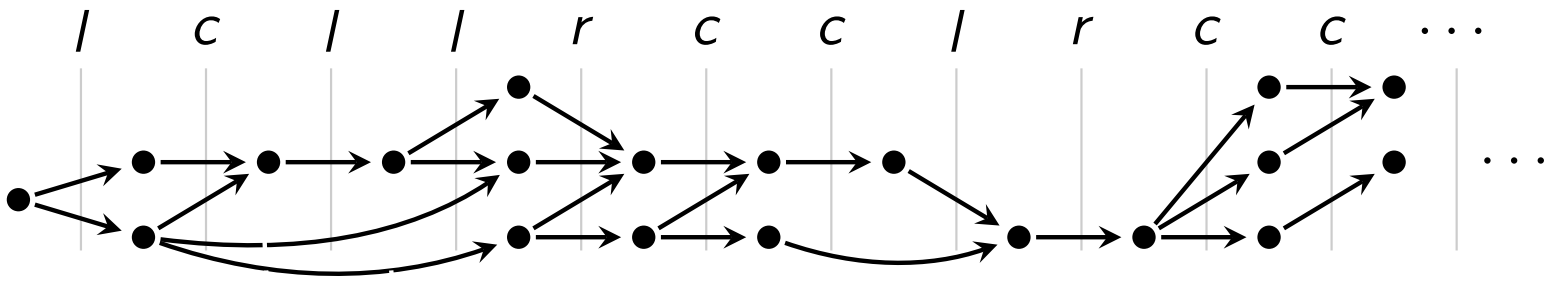
Acceptance: Every branch has infinitely many accepting vertices

Component	Technique
States	?
Transitions	?
Accepting States	?

Translating 1 – AJAs into Tree Automata

$$/ \quad c \quad / \quad / \quad r \quad c \quad c \quad / \quad r \quad c \quad c \quad \cdots$$

Translating 1 – AJAs into Tree Automata



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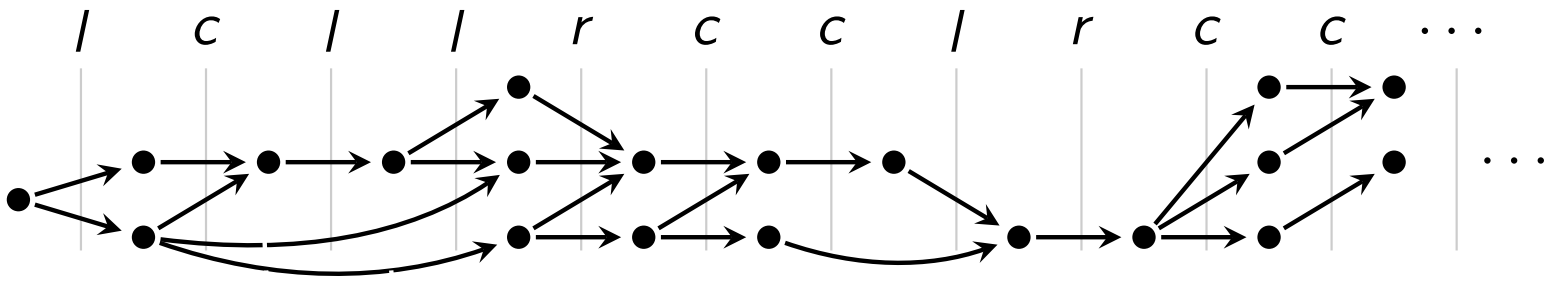

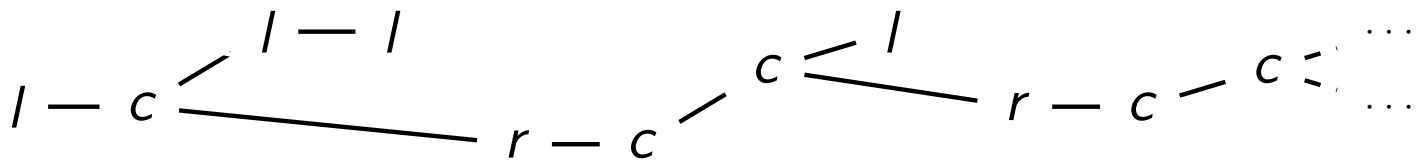

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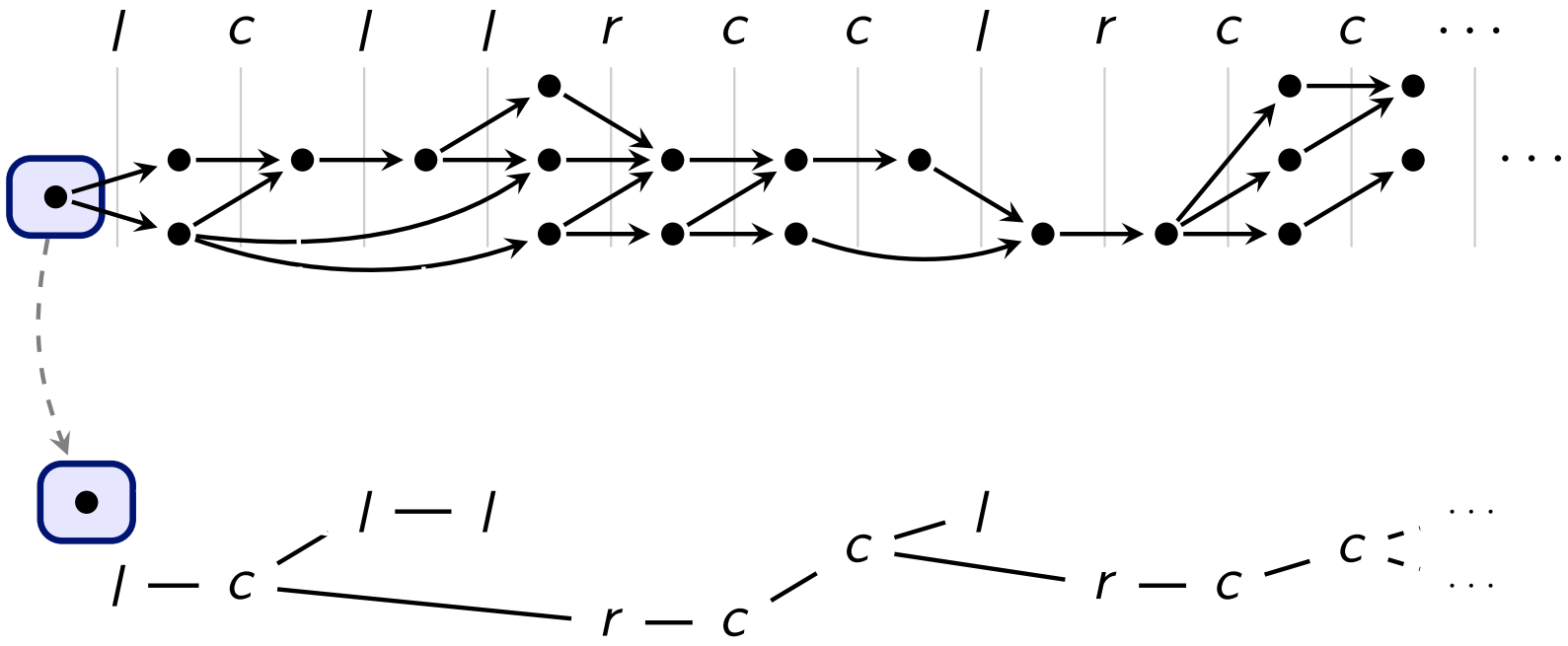

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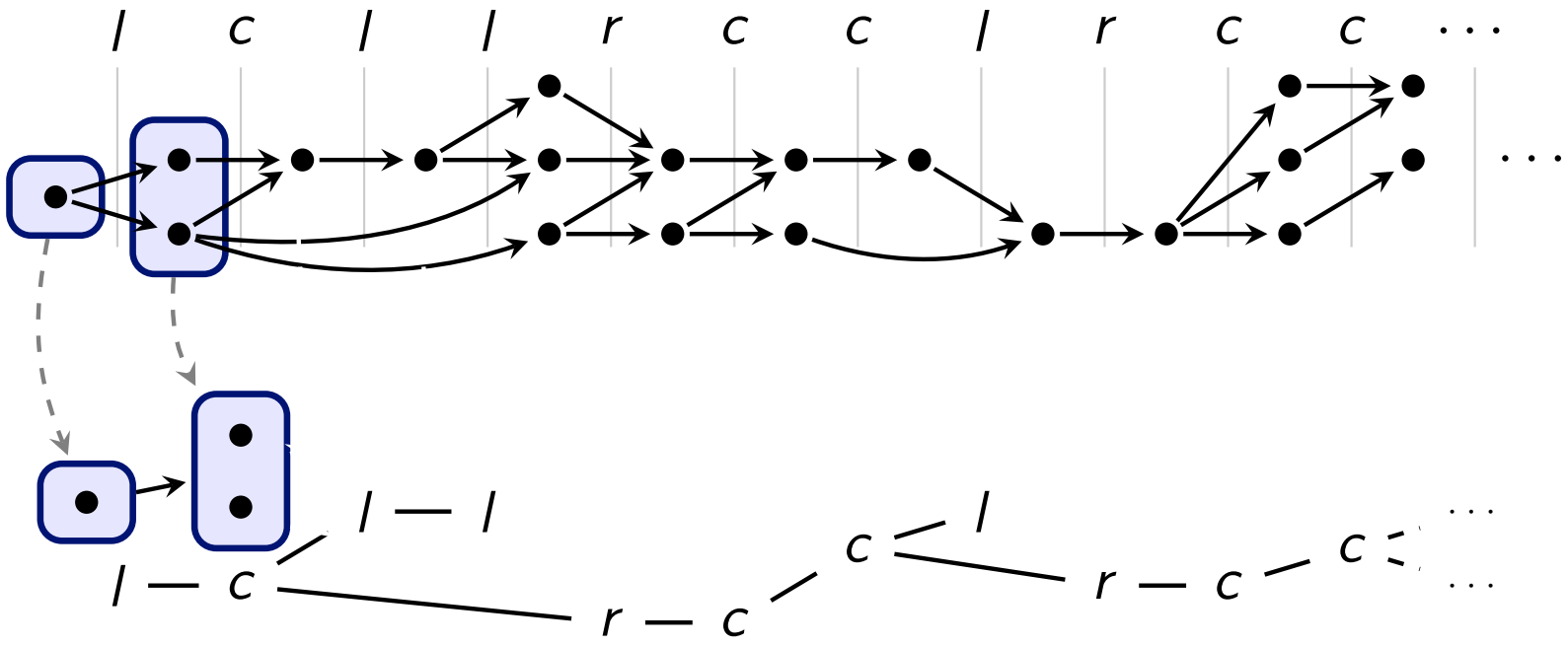

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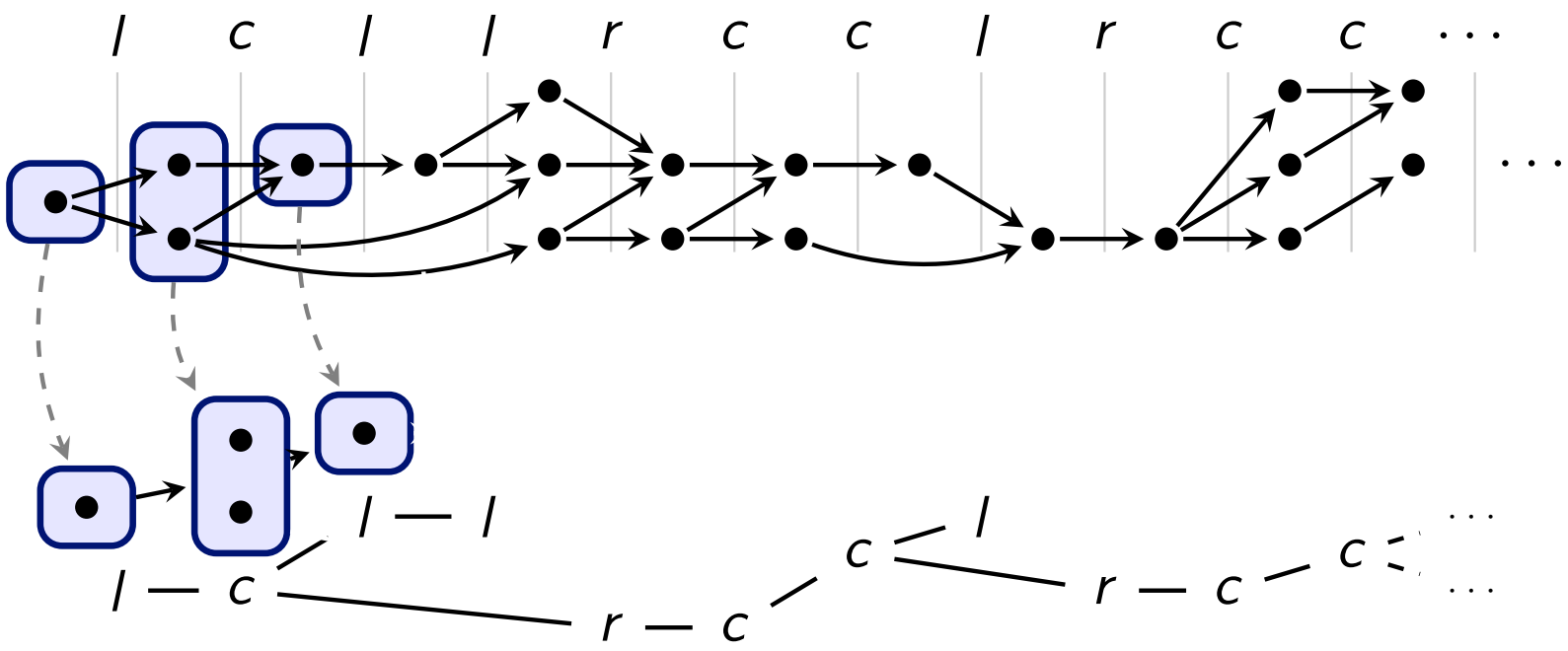
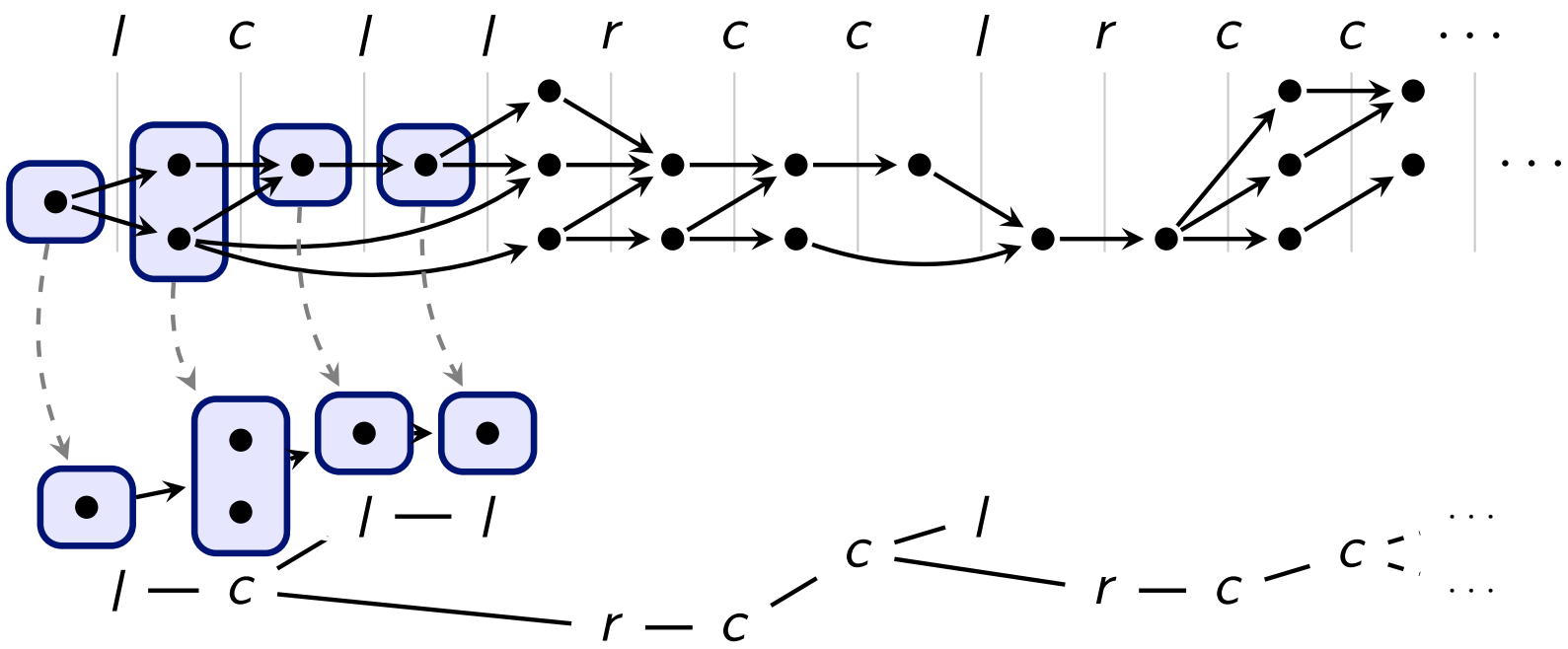


Translating 1 – AJAs into Tree Automata



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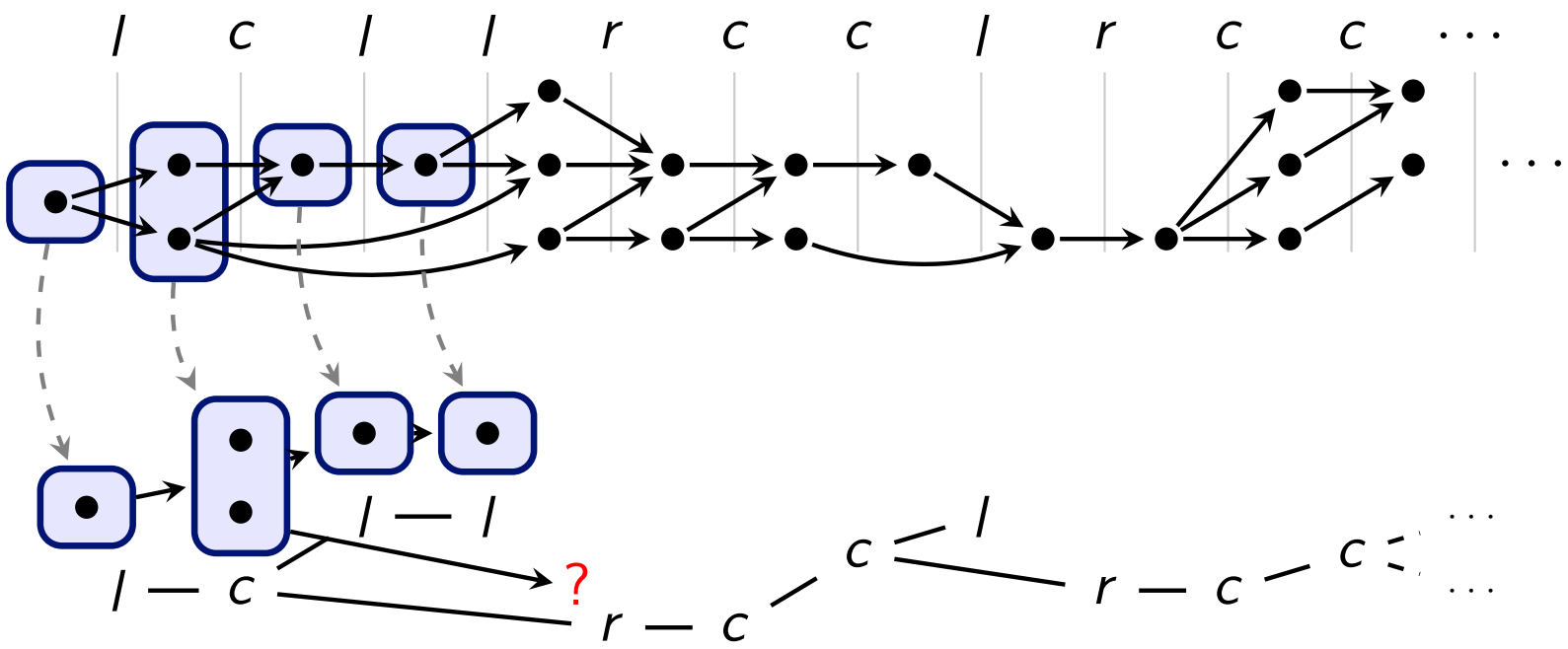
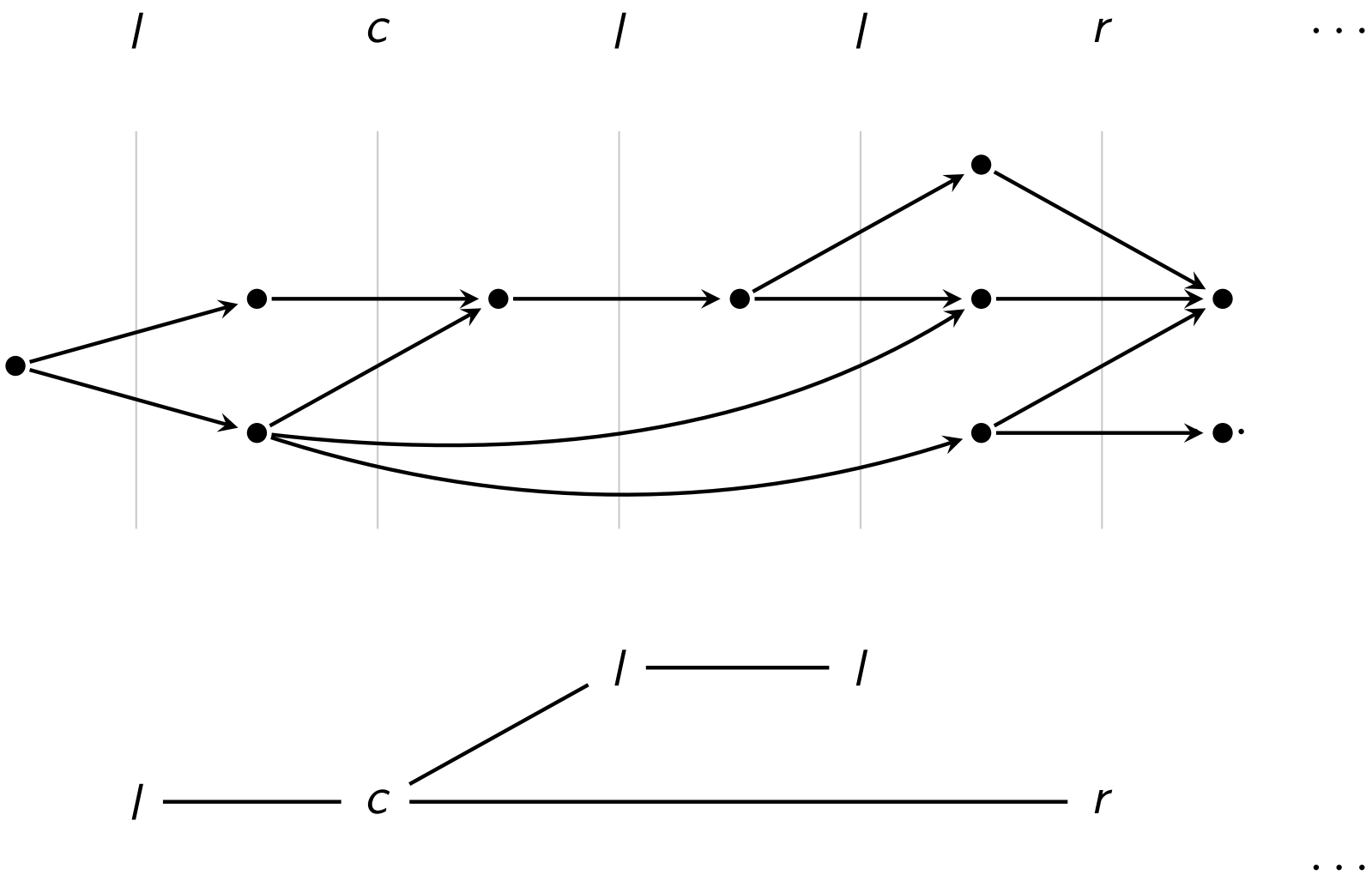


Construction in Detail



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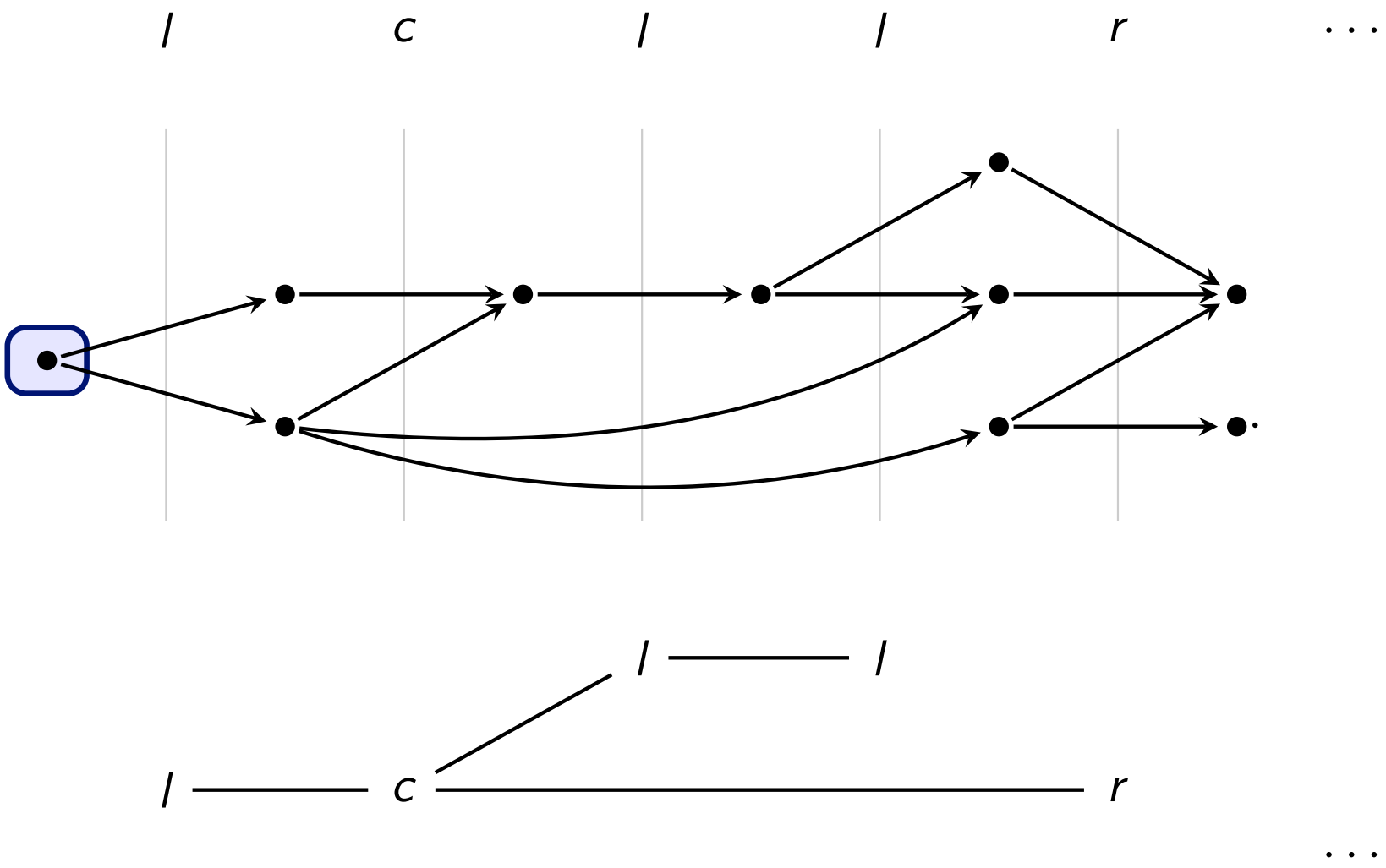

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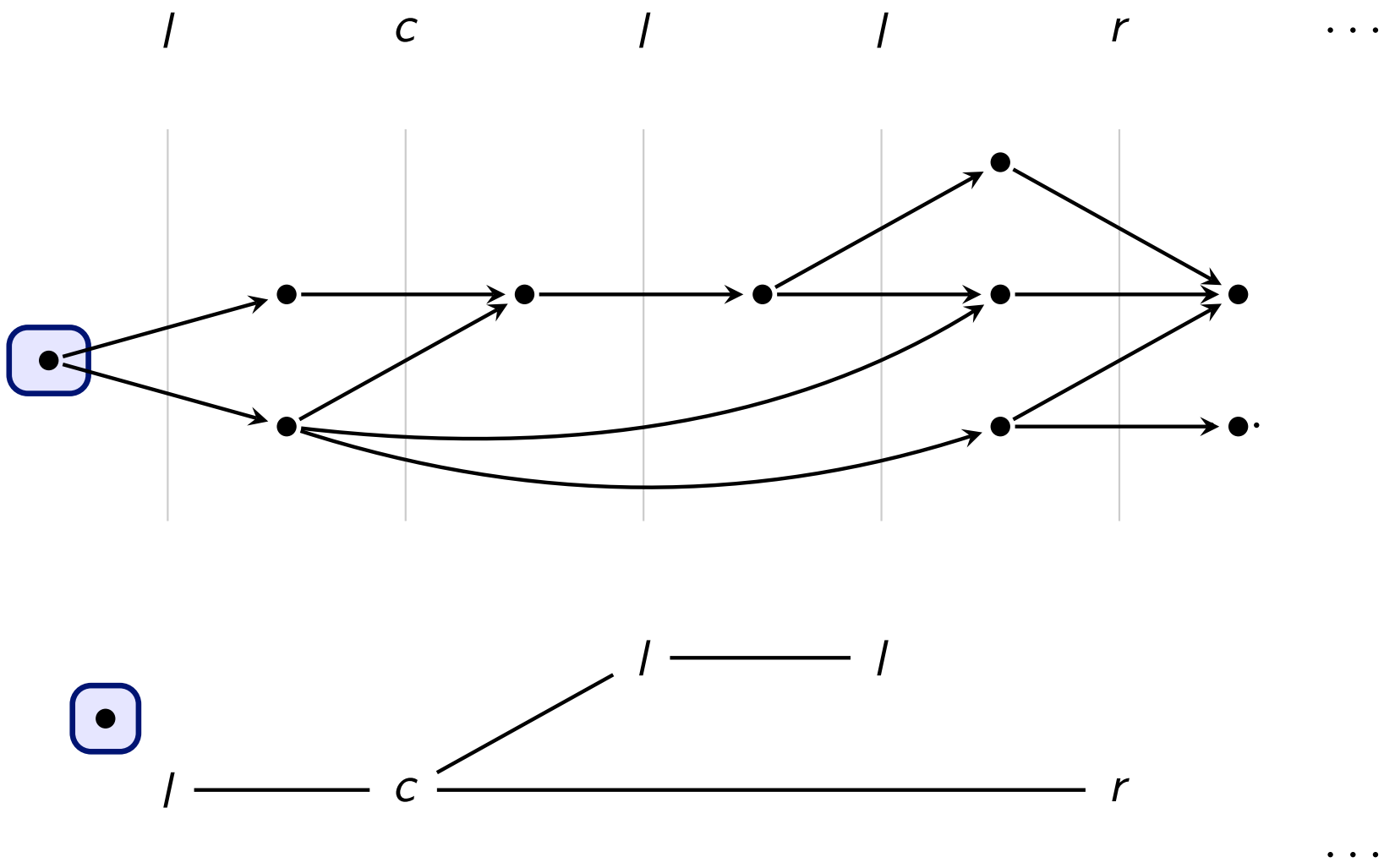

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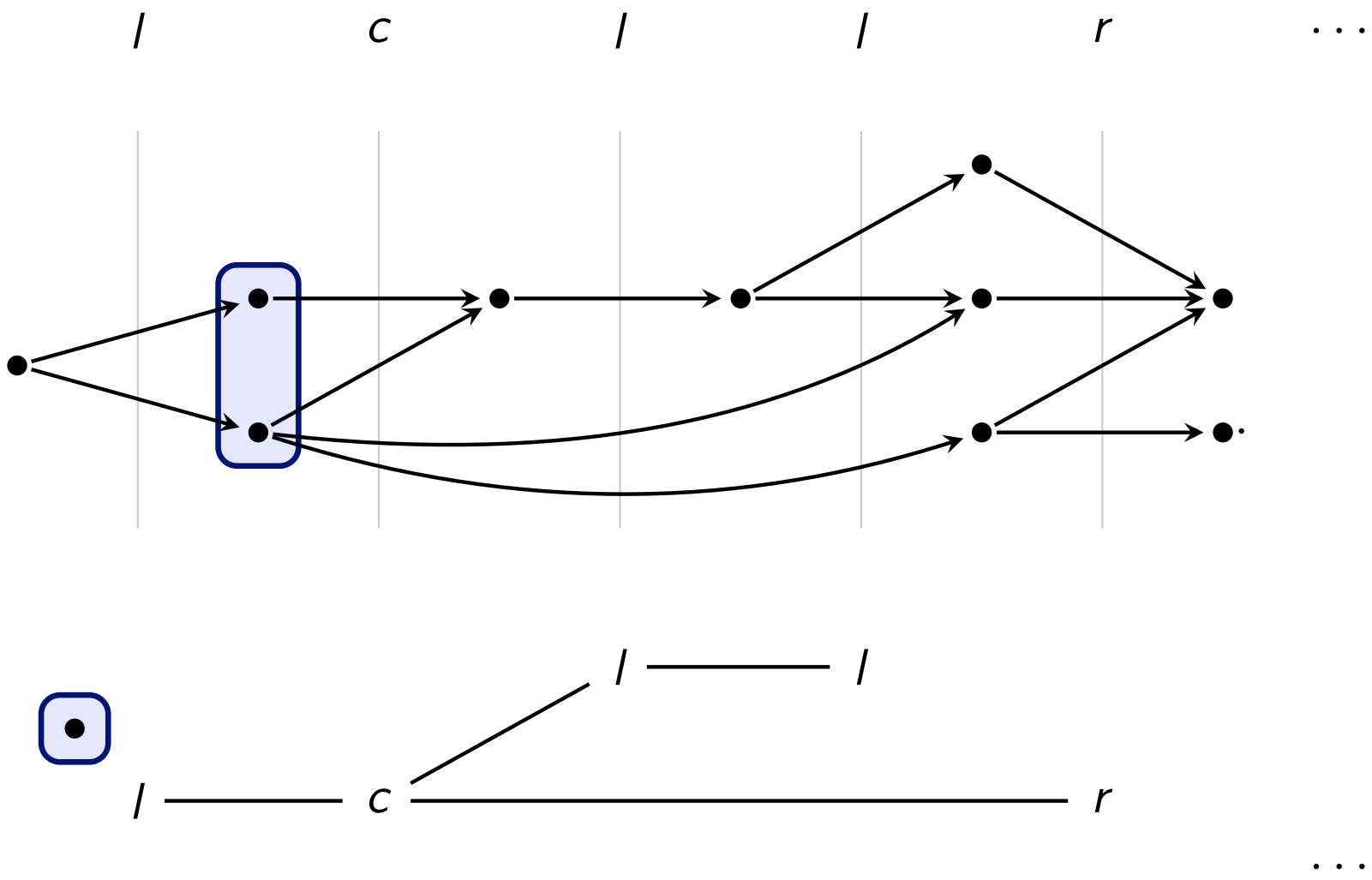

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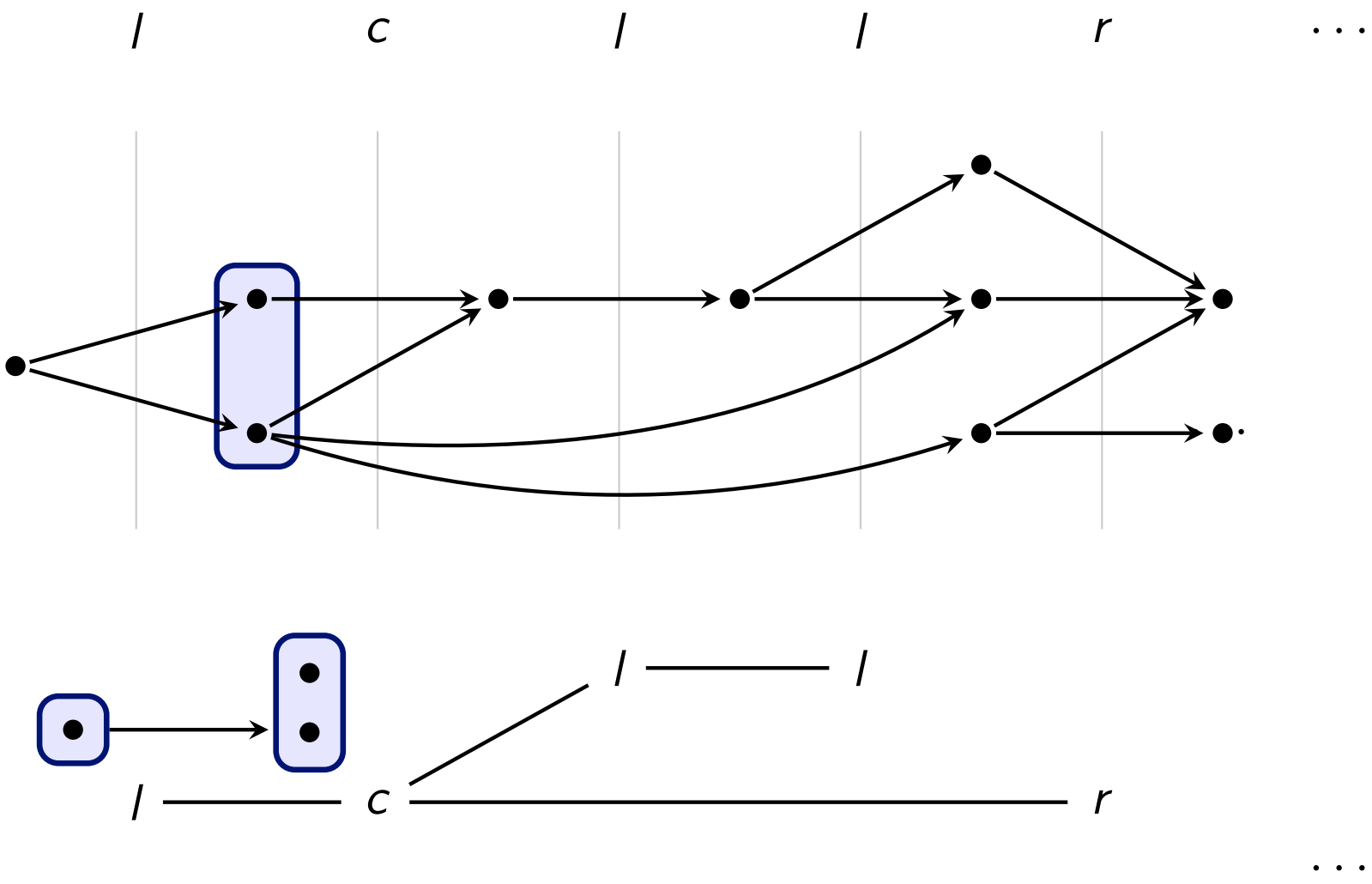

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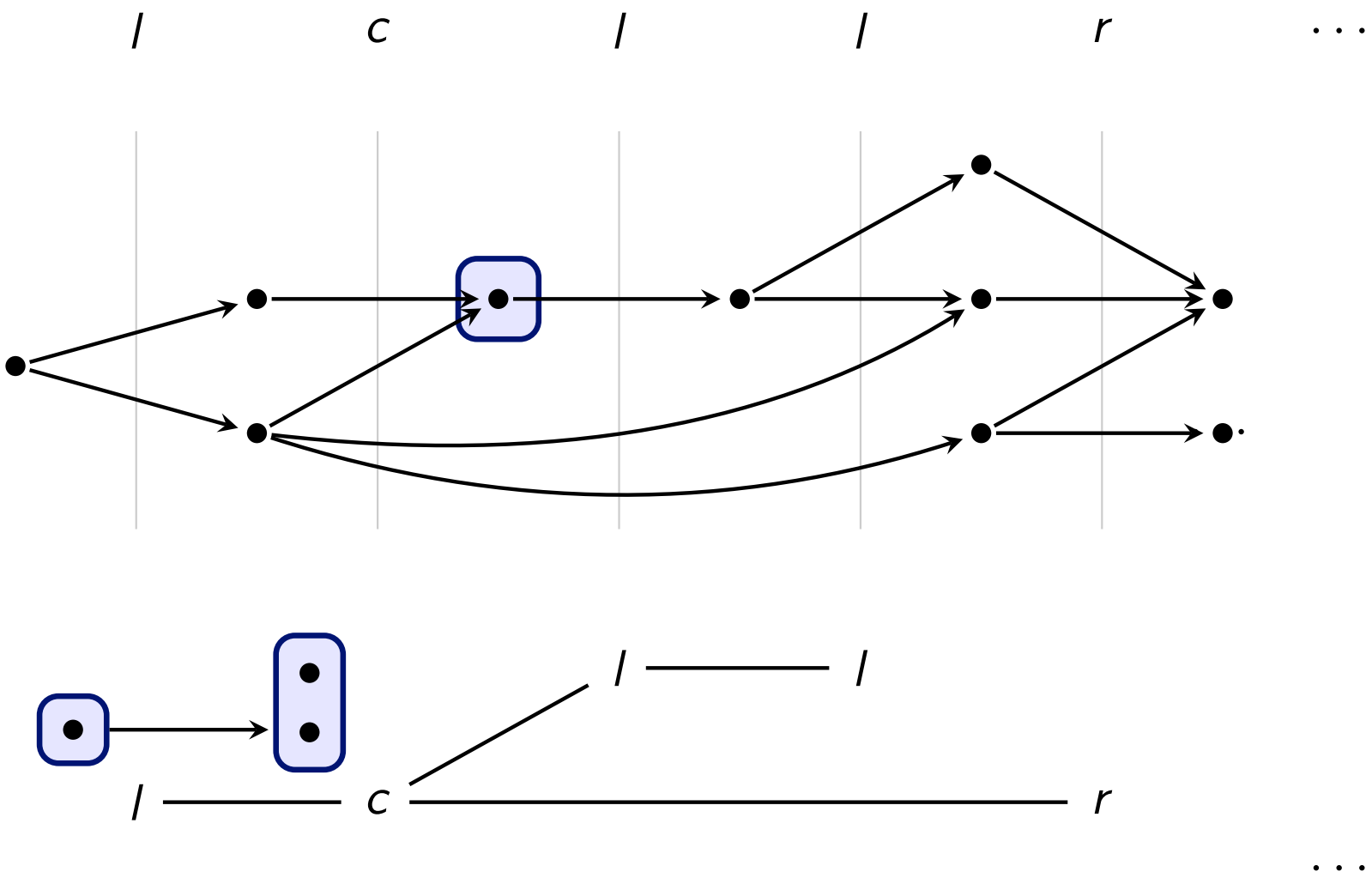

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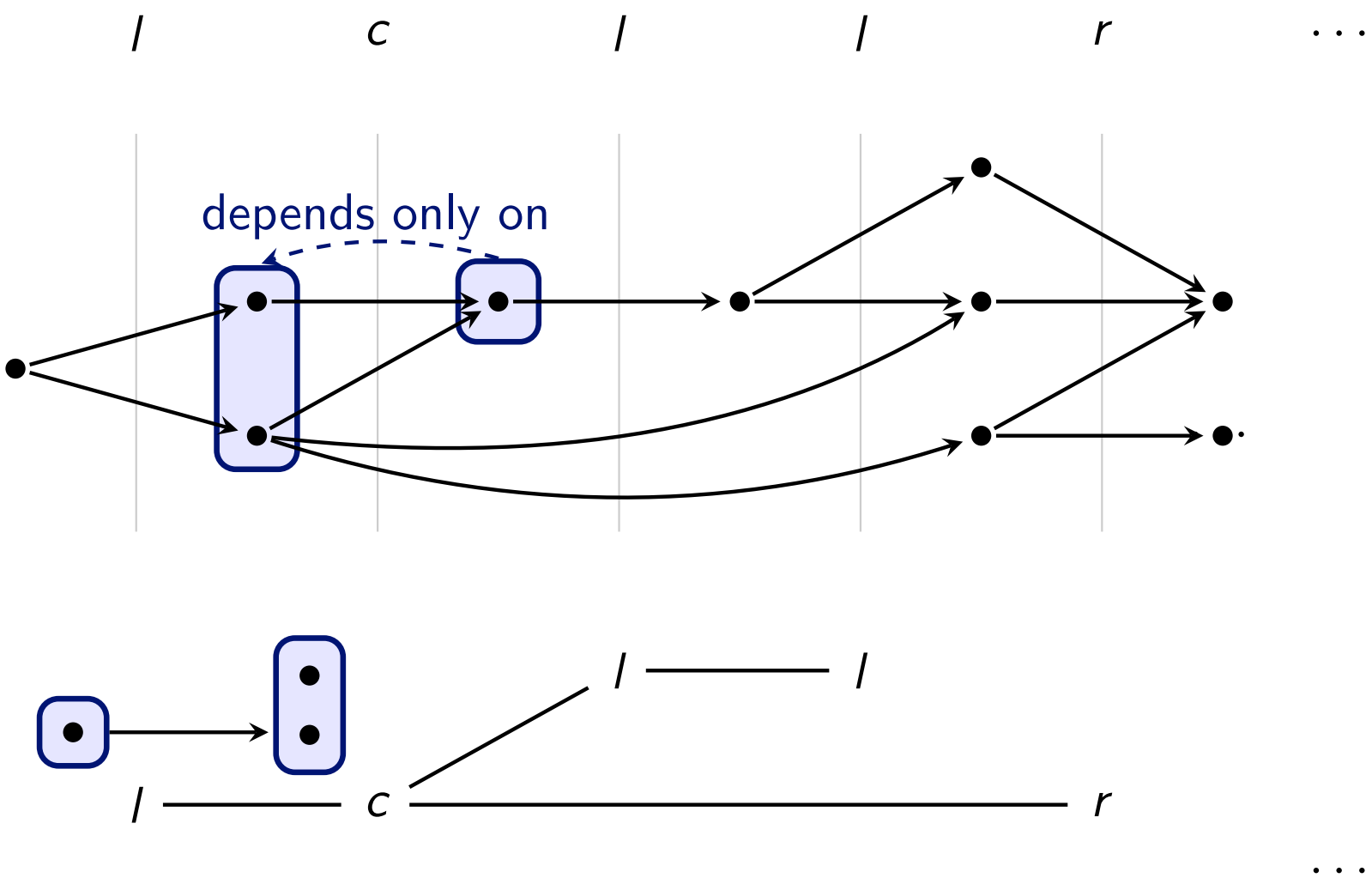

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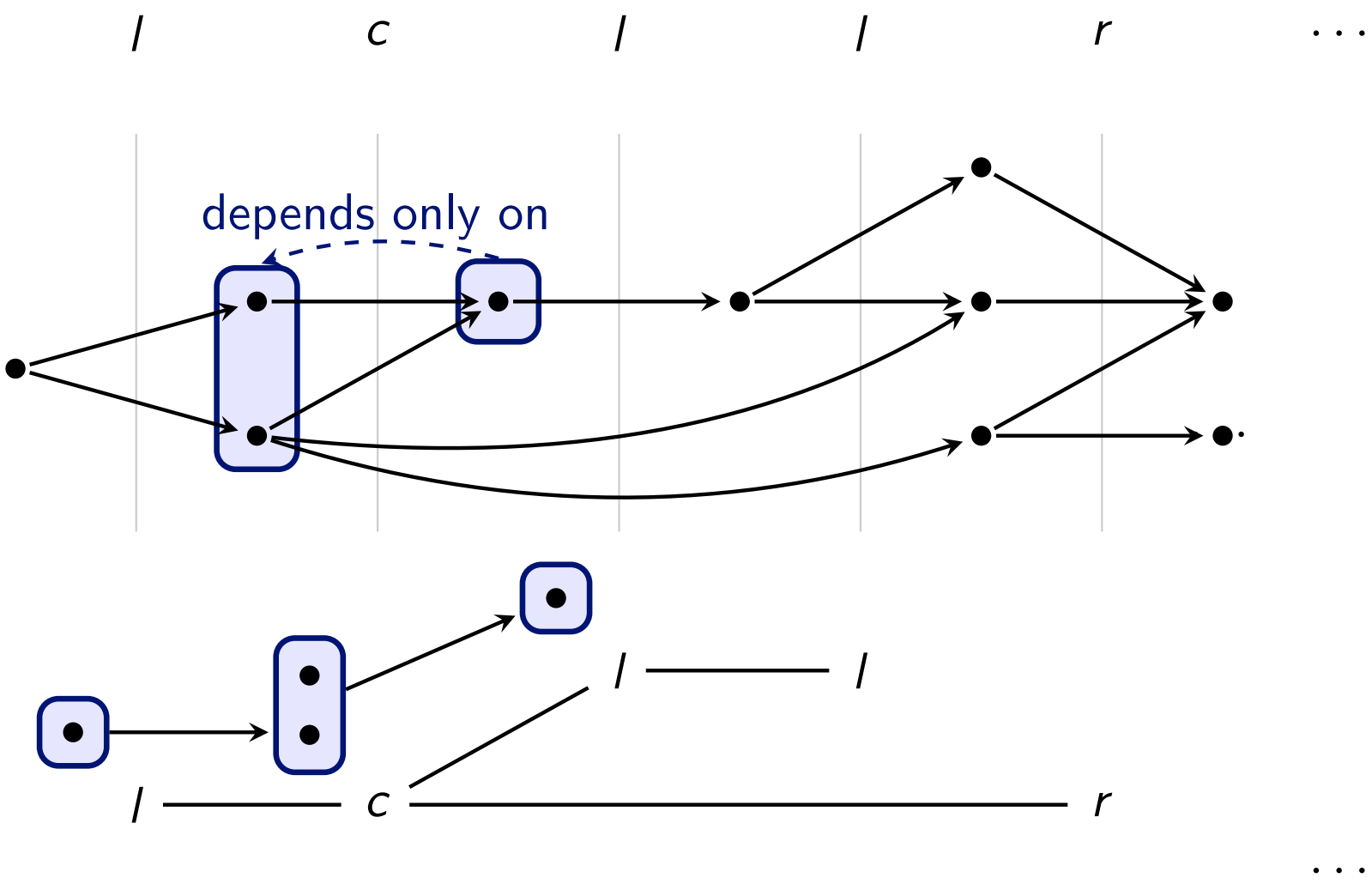

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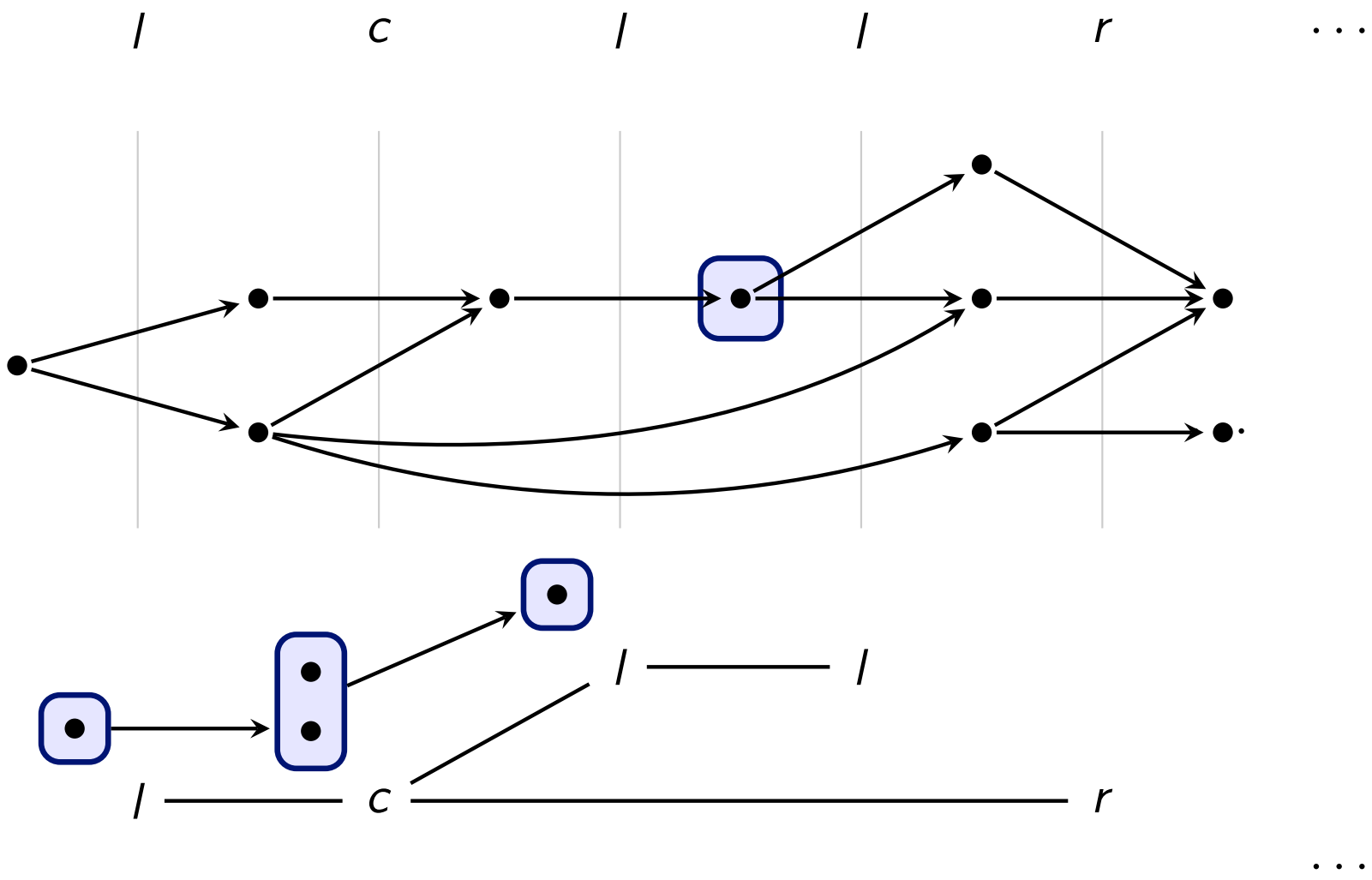

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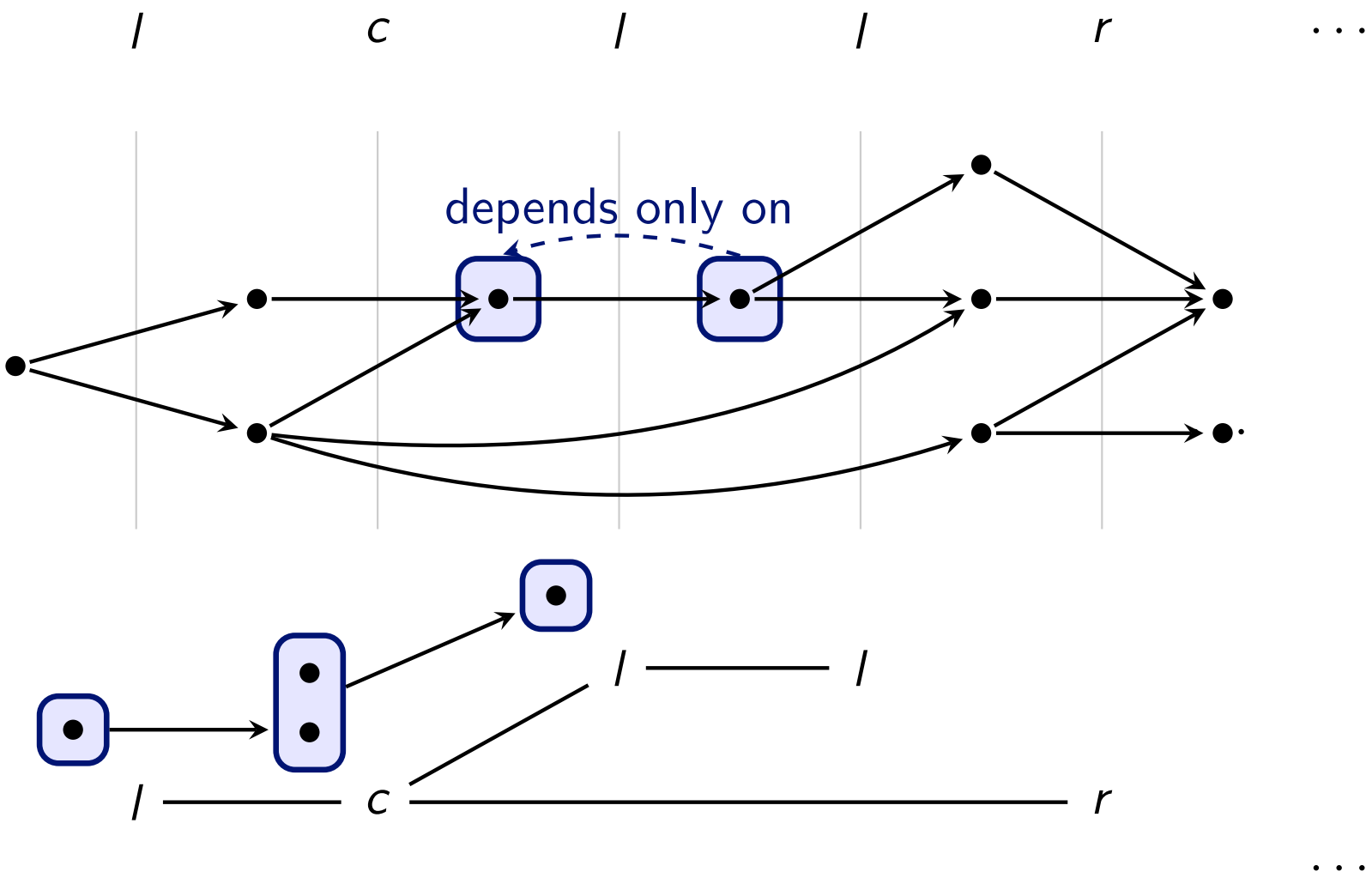

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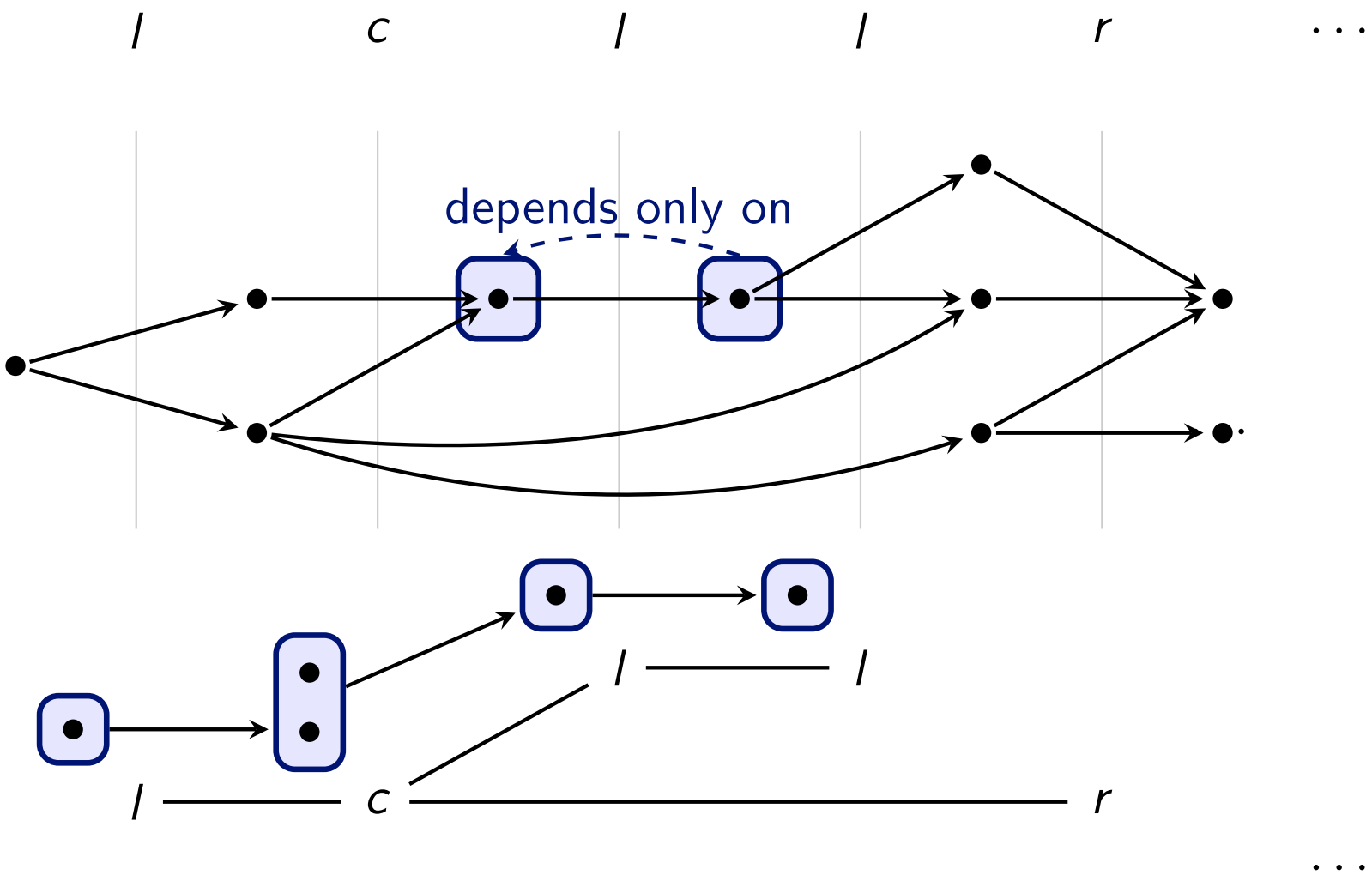

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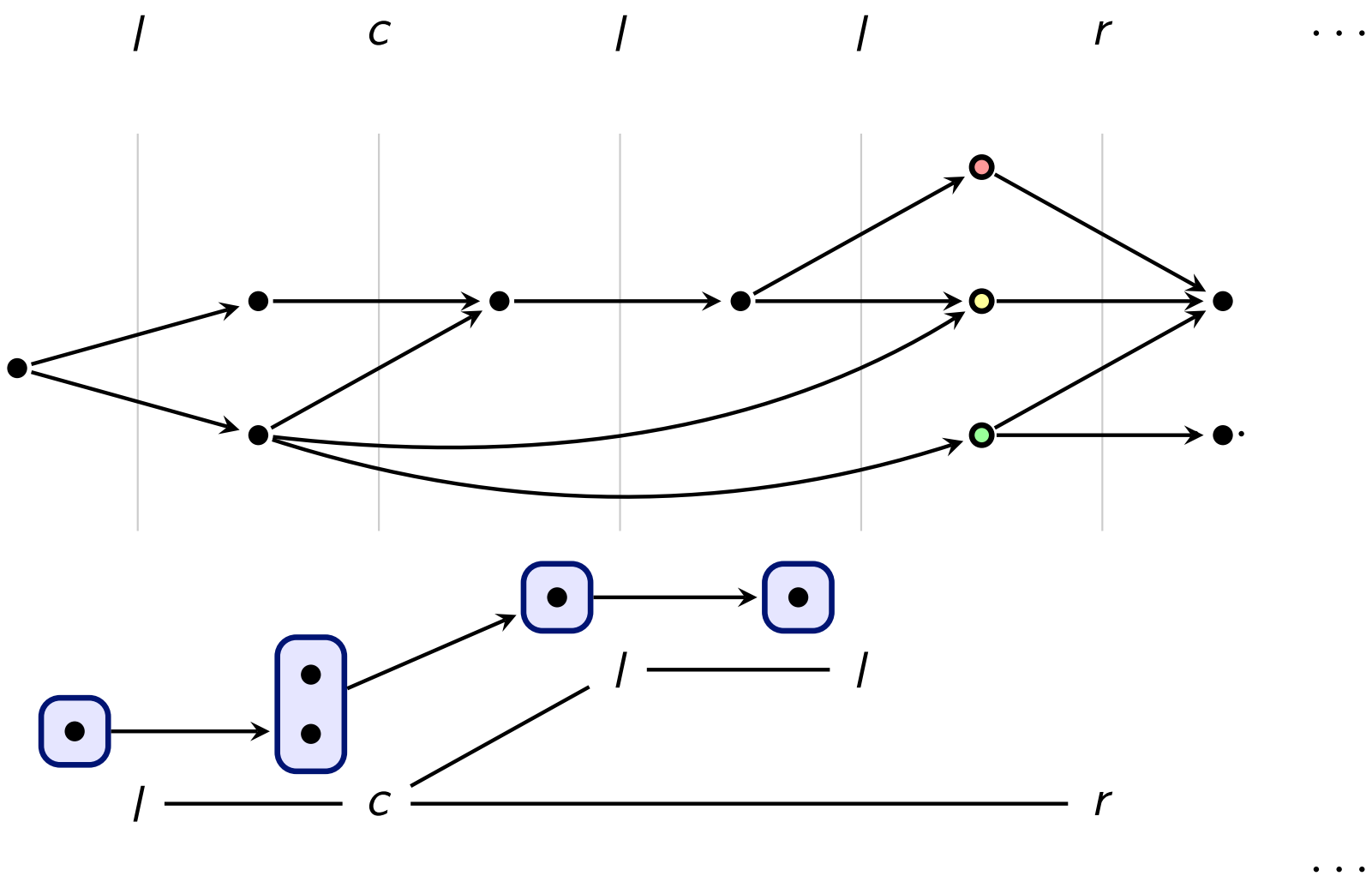

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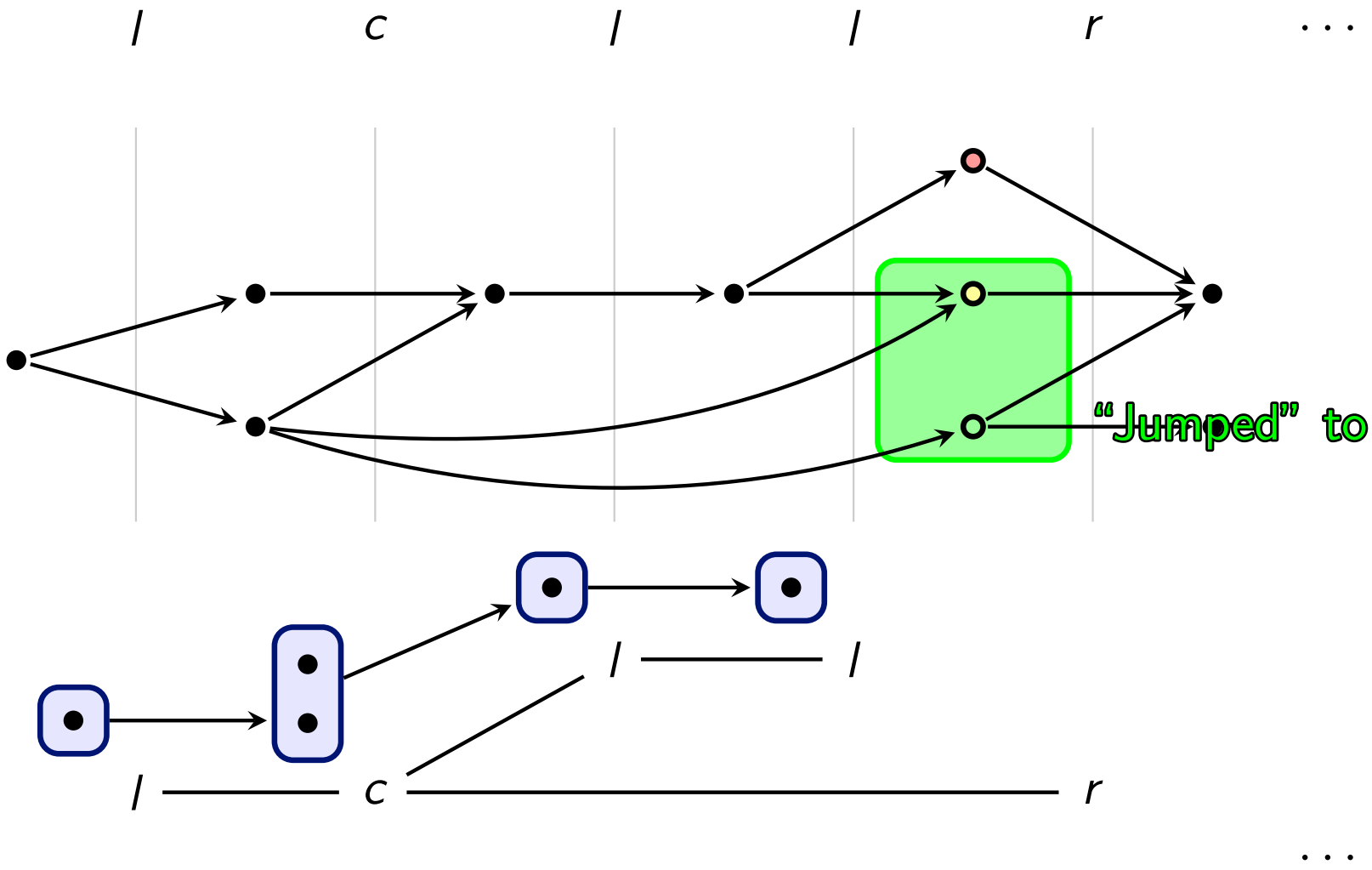

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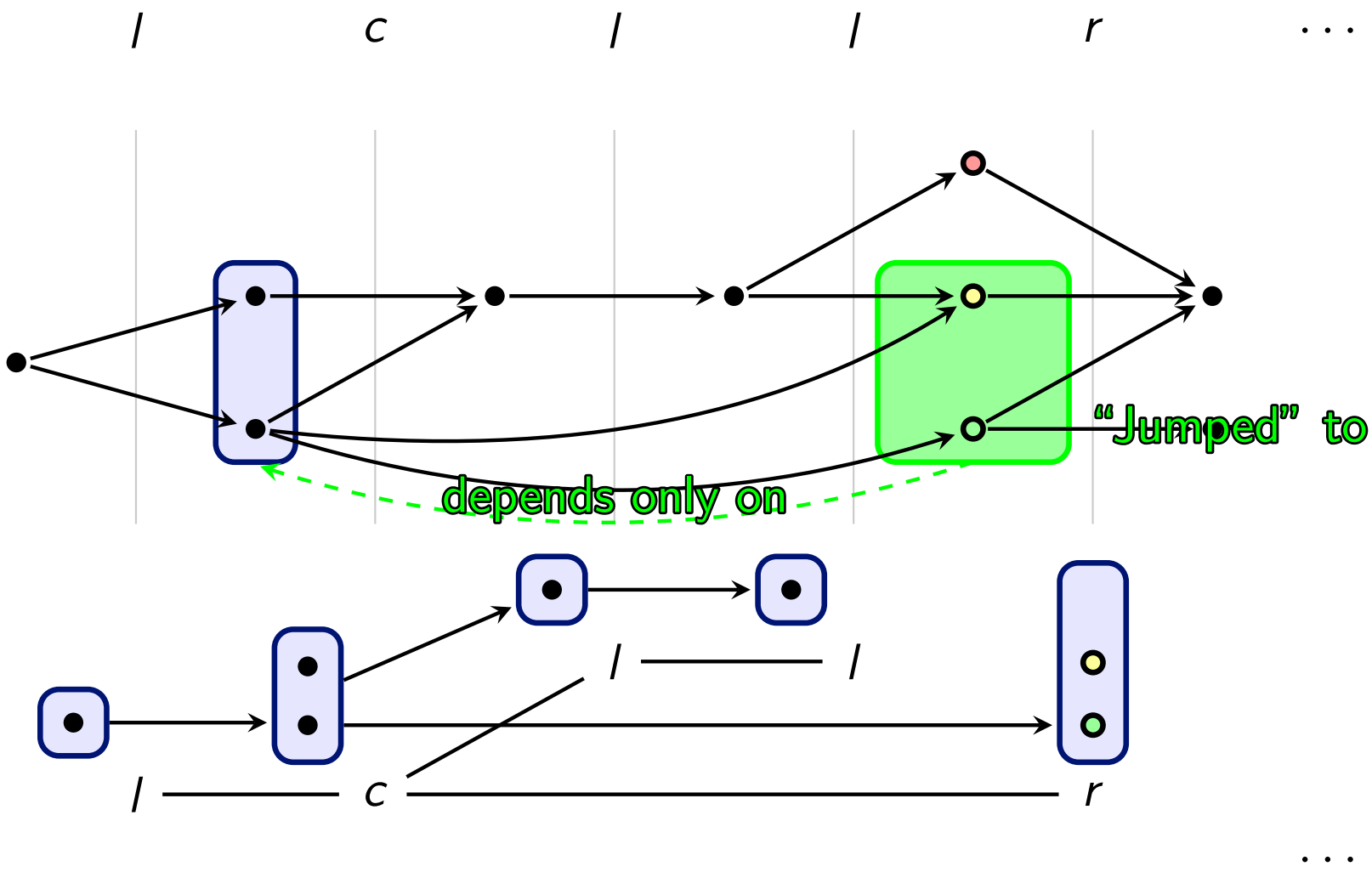

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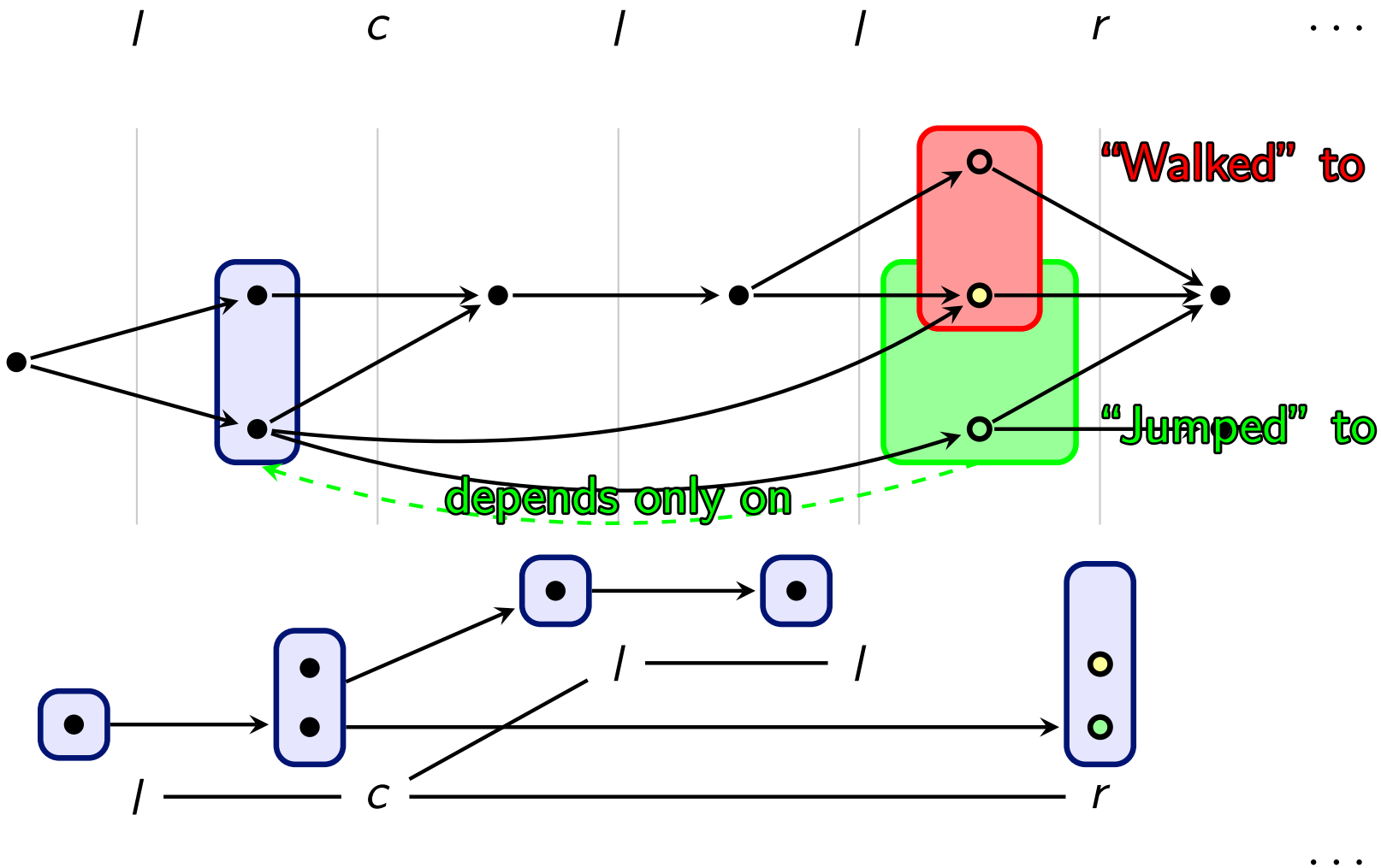

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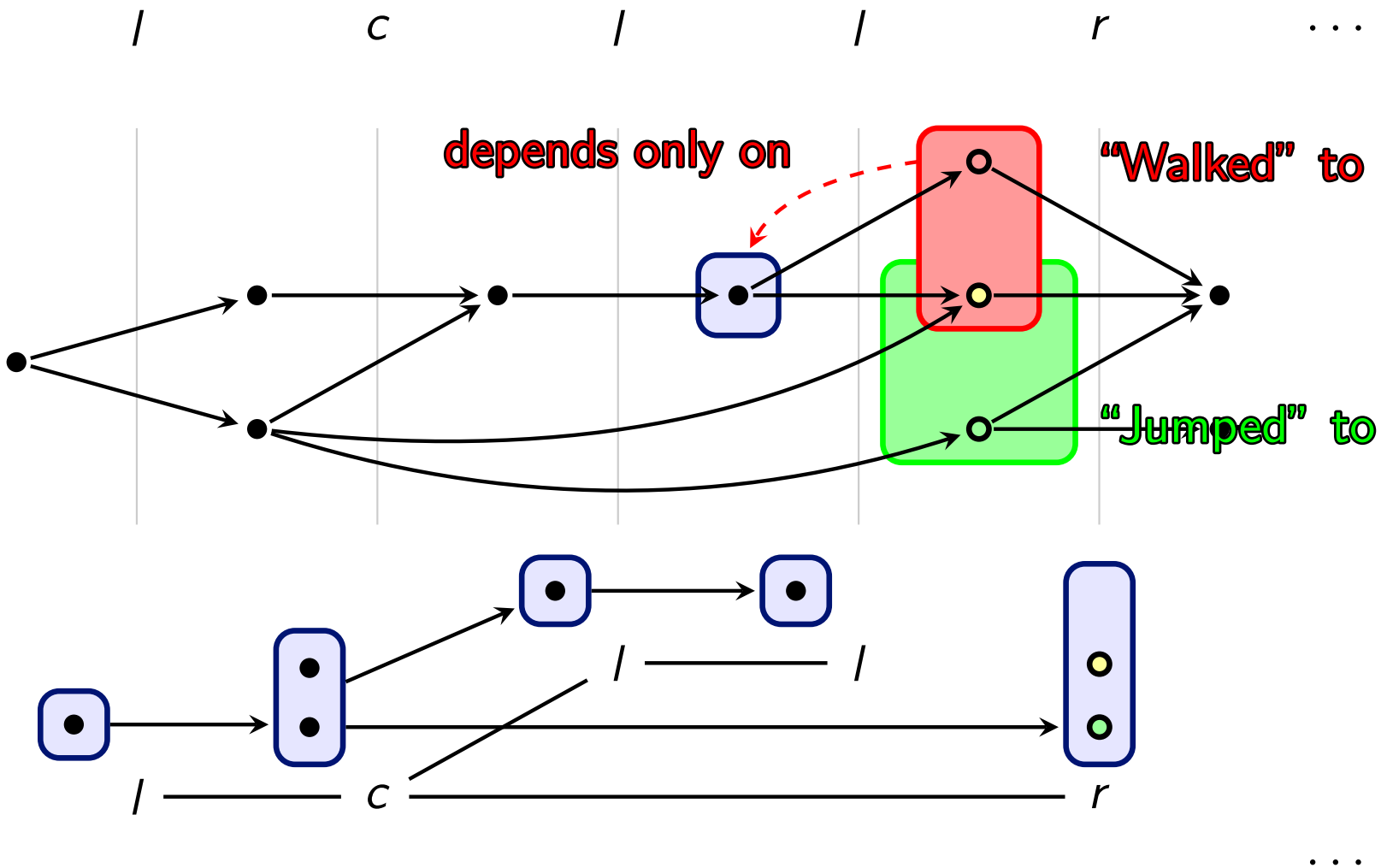

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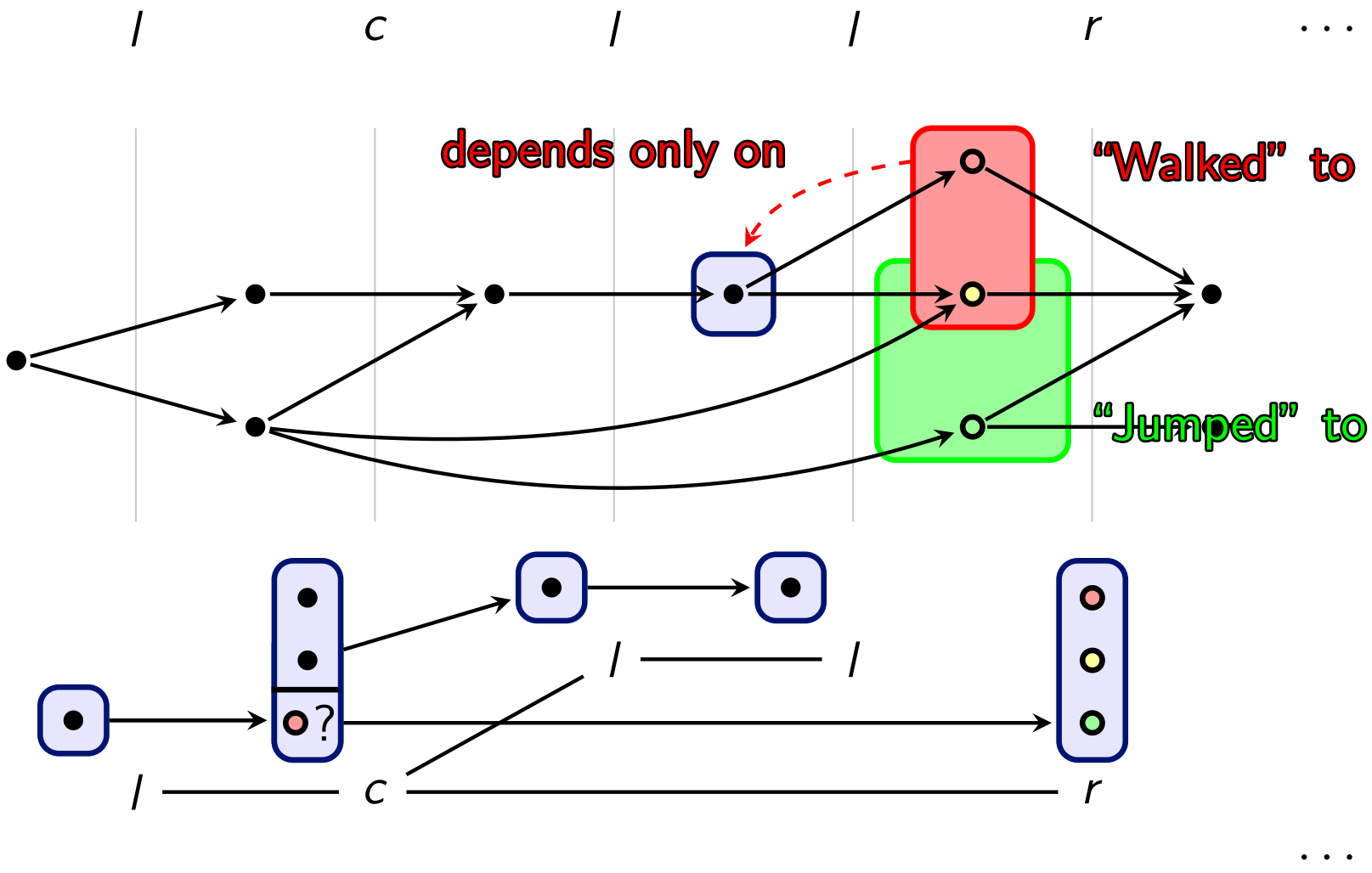

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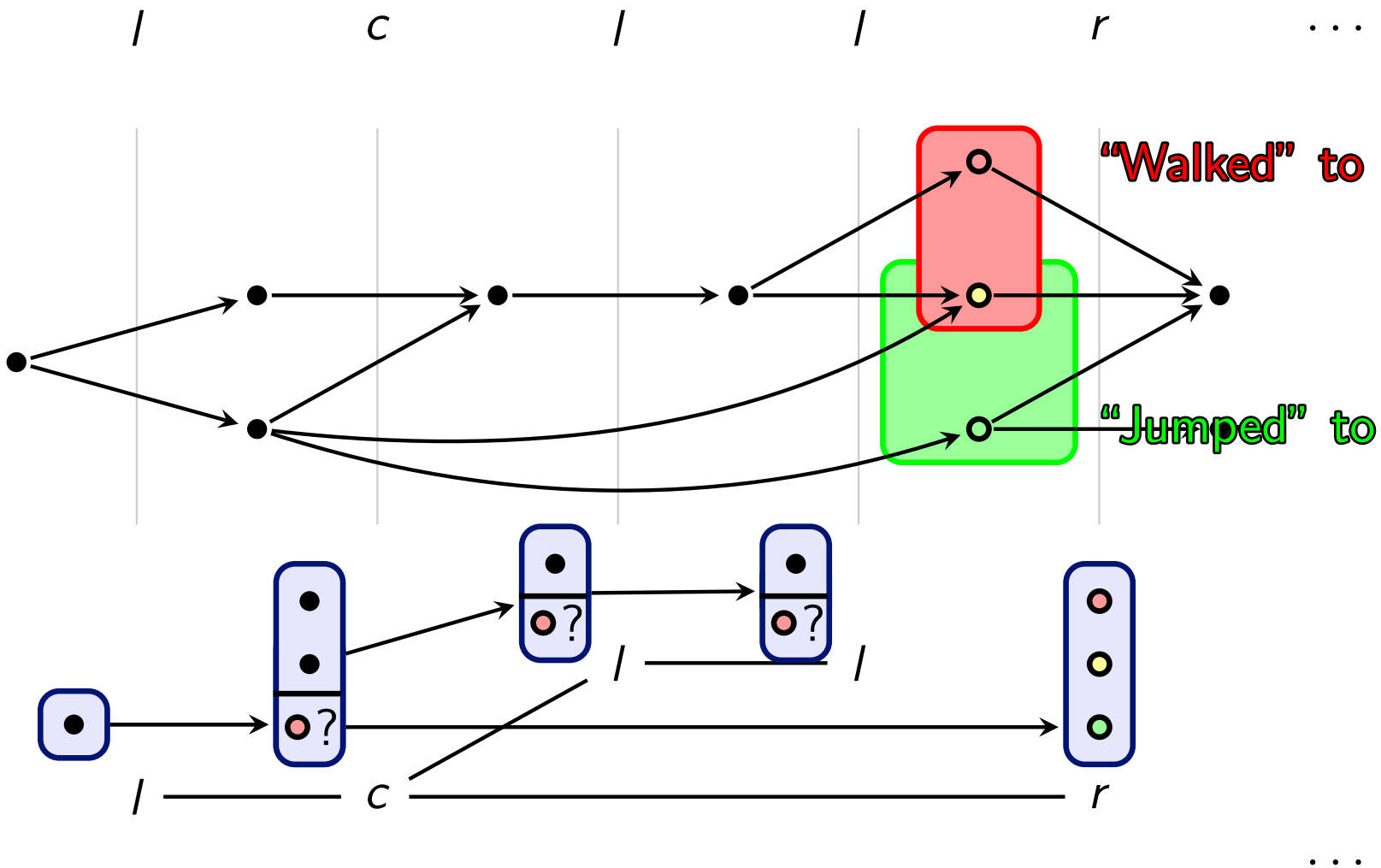
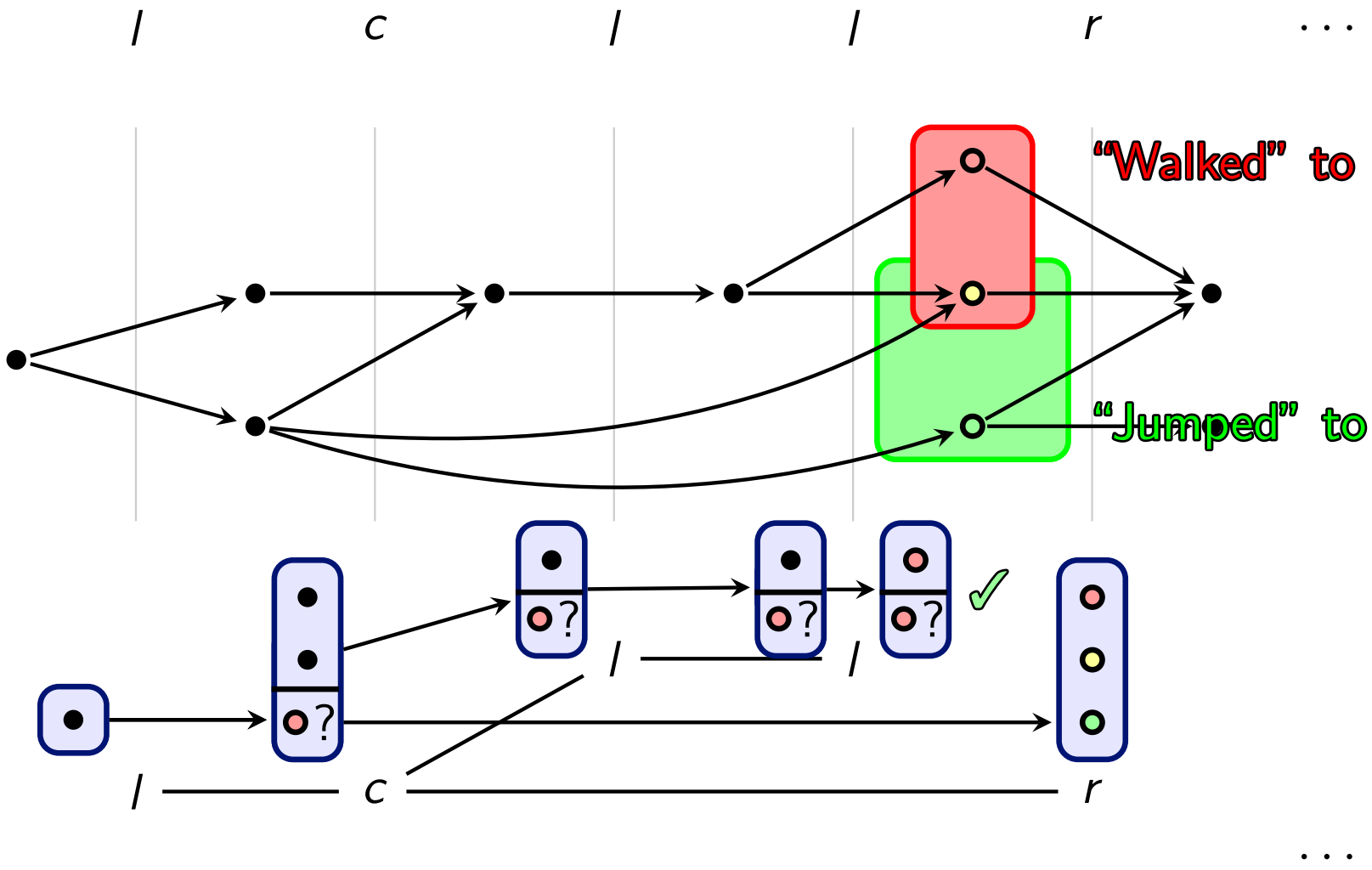
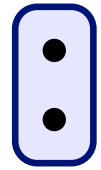


Construction in Detail



Summary of Construction

1. Subset construction



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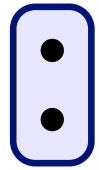
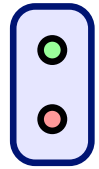


2. Split into “jump”- and “walk”-states



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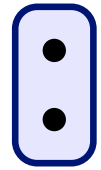

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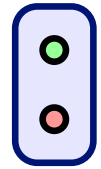


3. Guess and verify

Constructing a Tree Automaton

Component	Technique
States	
Transitions	
Accepting States	

Constructing a Tree Automaton

Component	Technique	
States	Subset Construction	✓
Transitions		
Accepting States		

Constructing a Tree Automaton

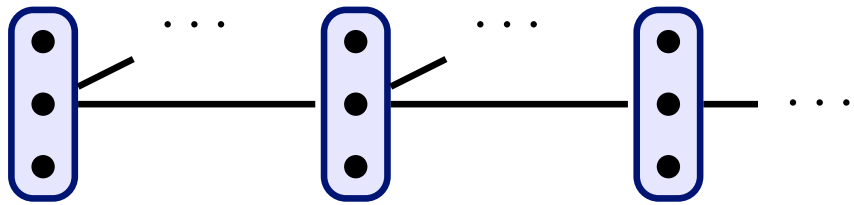
Component	Technique	
States	Subset Construction	✓
Transitions	Guess/Verify	✓
Accepting States		

Constructing a Tree Automaton

Component	Technique	
States	Subset Construction	✓
Transitions	Guess/Verify	✓
Accepting States	?	

Accepting States

Run of the tree automaton:



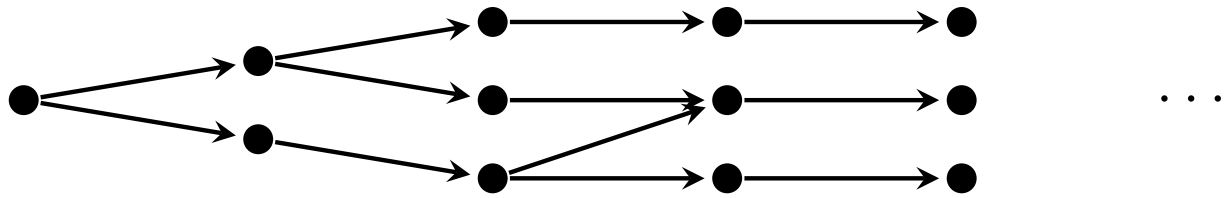
Recall:

- $1 - \mathrm{AJA}$ accepts if all paths are accepting.
- Tree automaton accepts if accepting states are visited infinitely often.

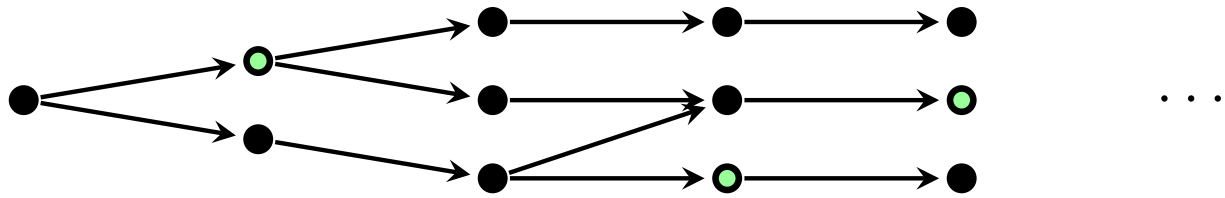
Solution: Lift acceptance condition from paths to states

$\Rightarrow$ Breakpoint Technique (Miyano and Hayashi, 1984)

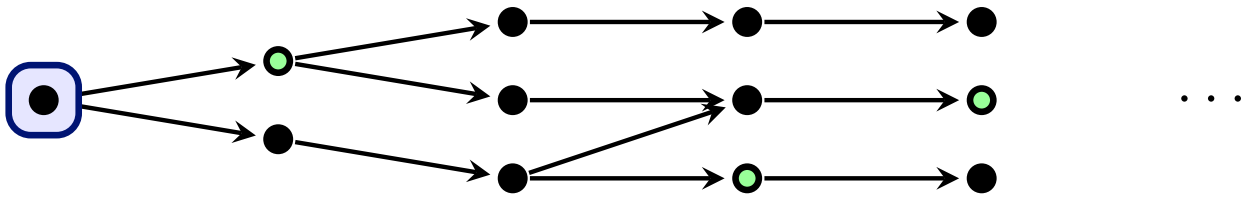
Breakpoint Construction



Breakpoint Construction

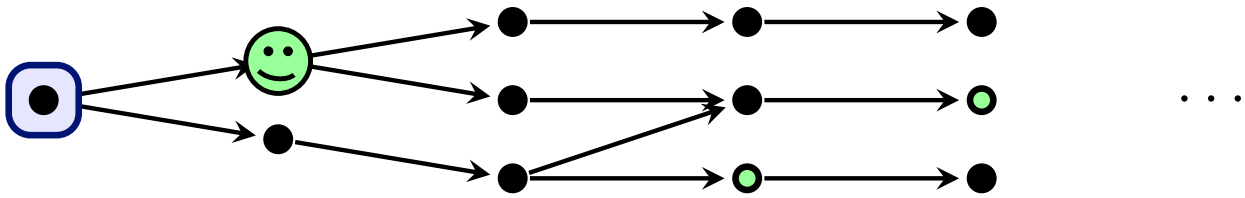


Breakpoint Construction

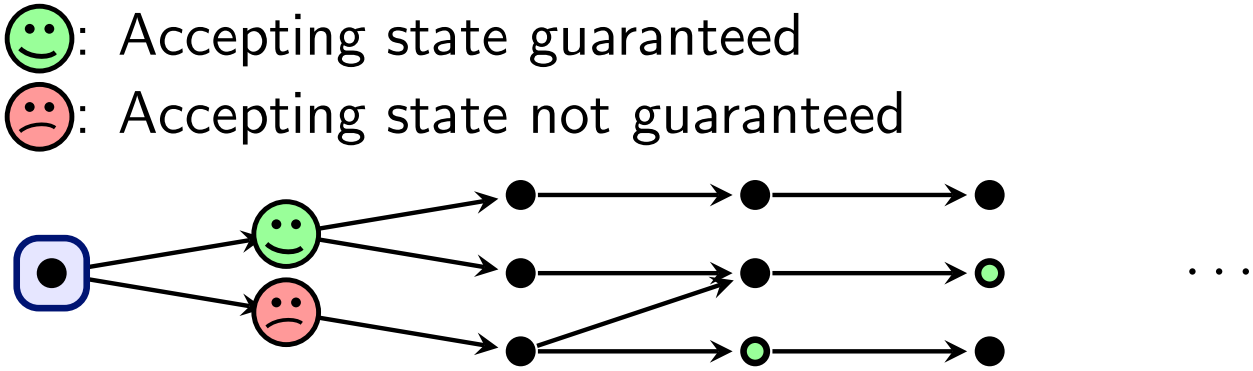


Breakpoint Construction

: Accepting state guaranteed



Breakpoint Construction



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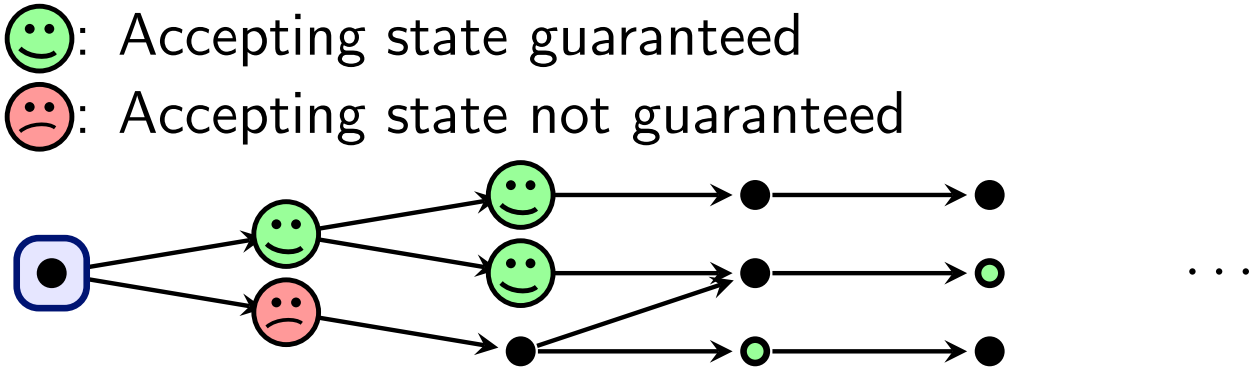

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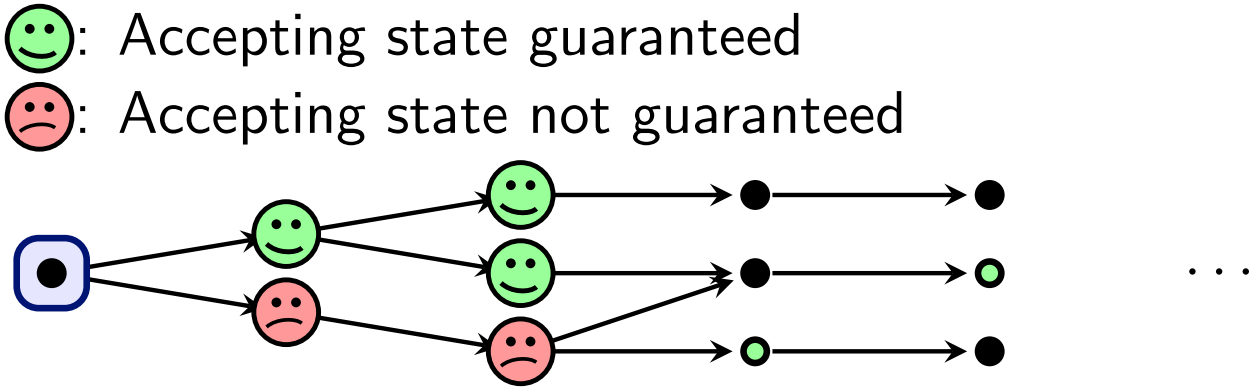

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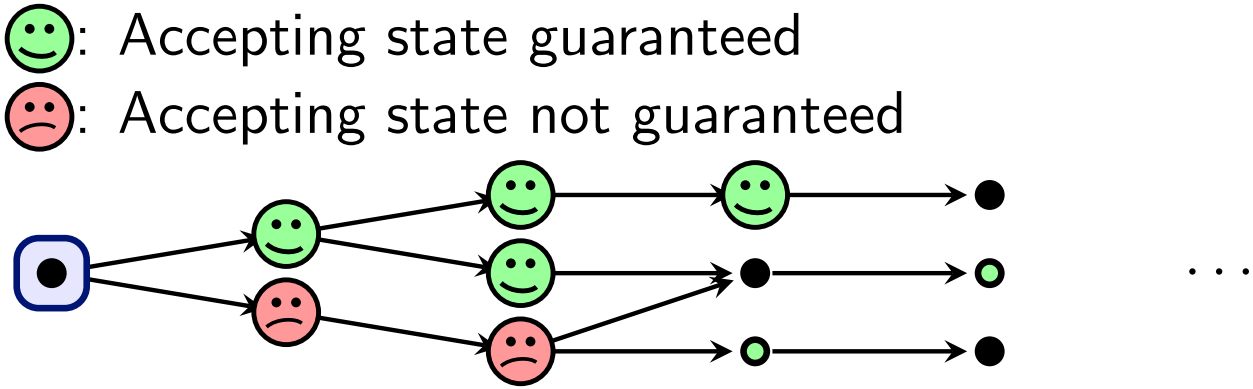

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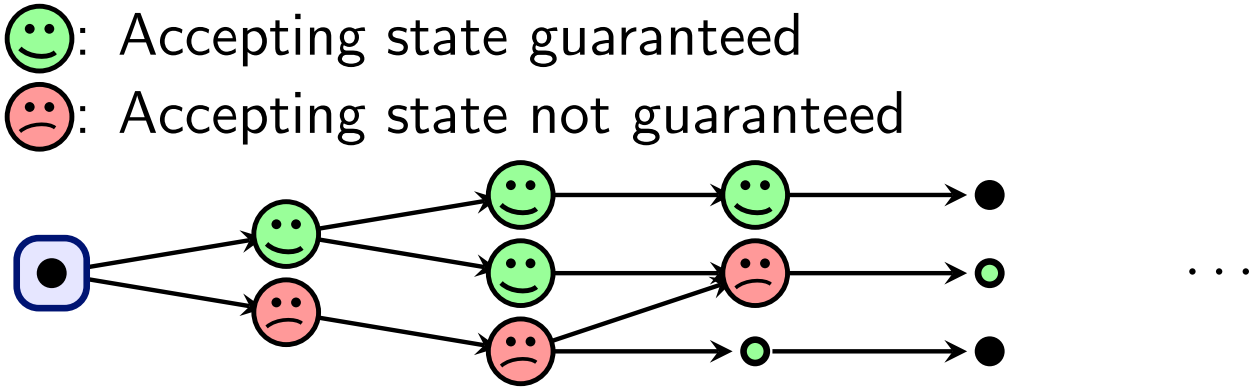

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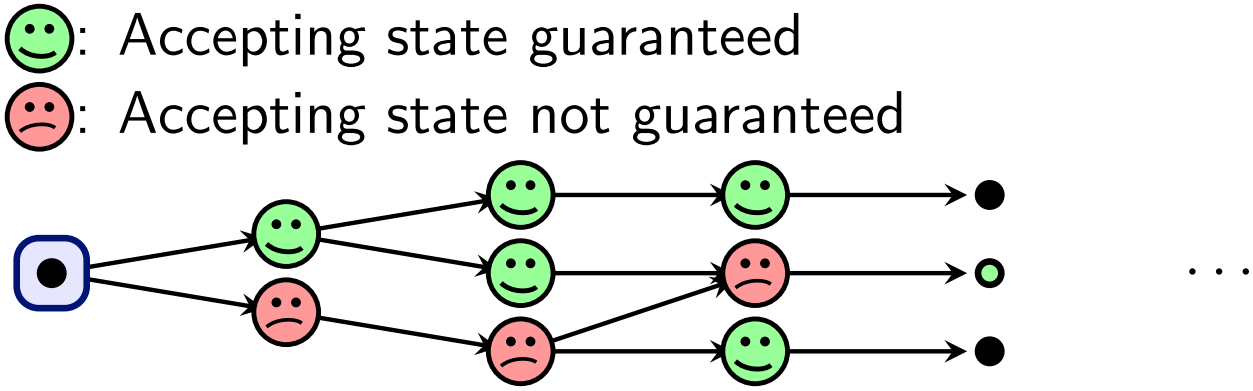

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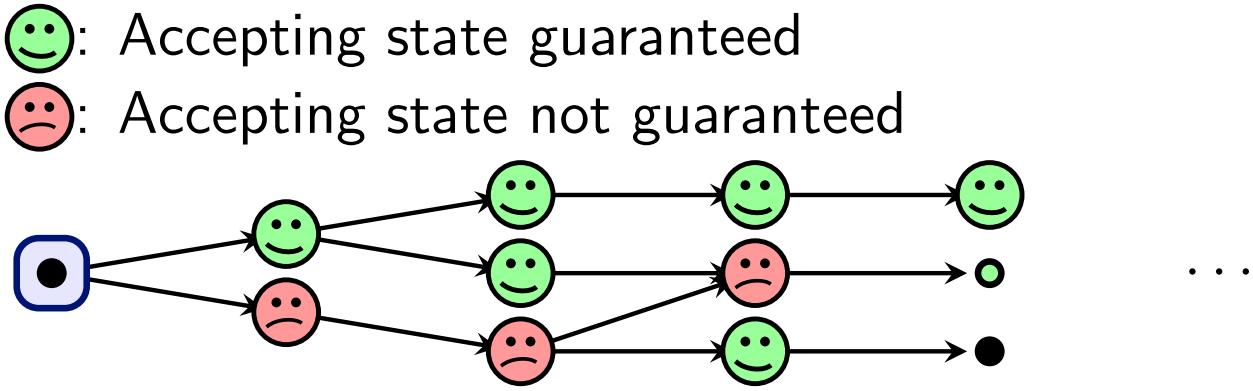

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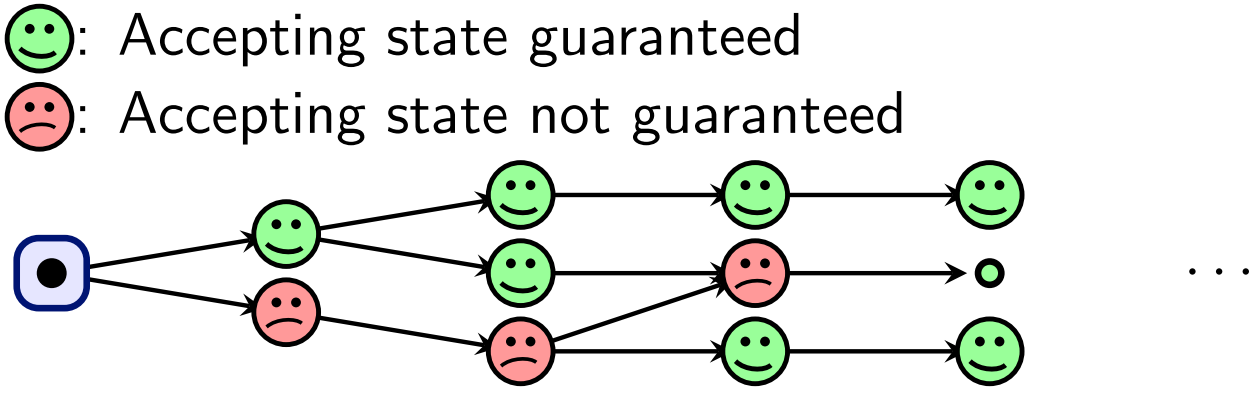

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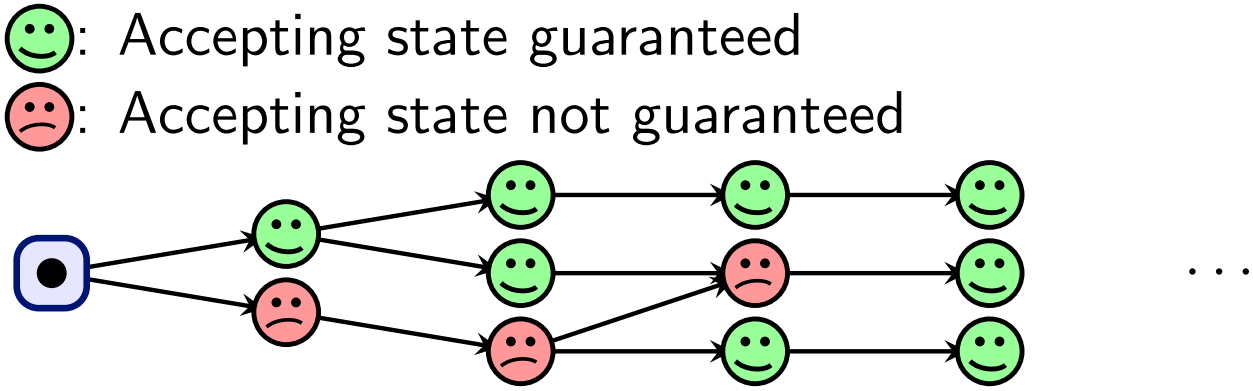

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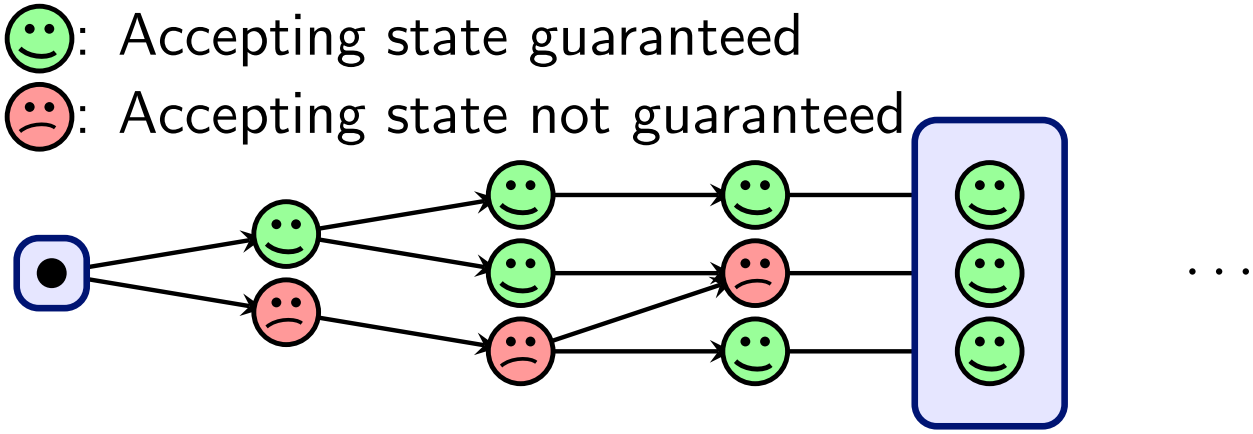

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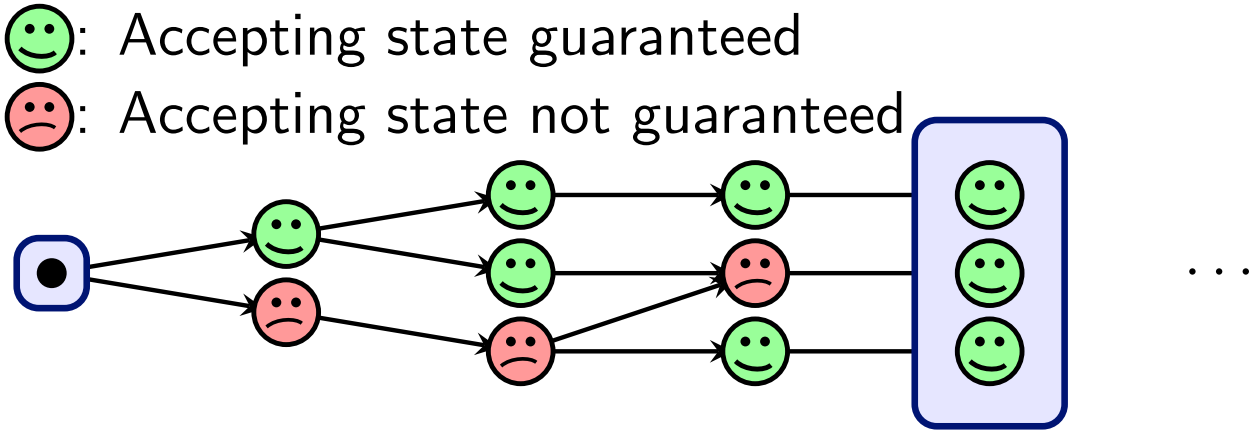
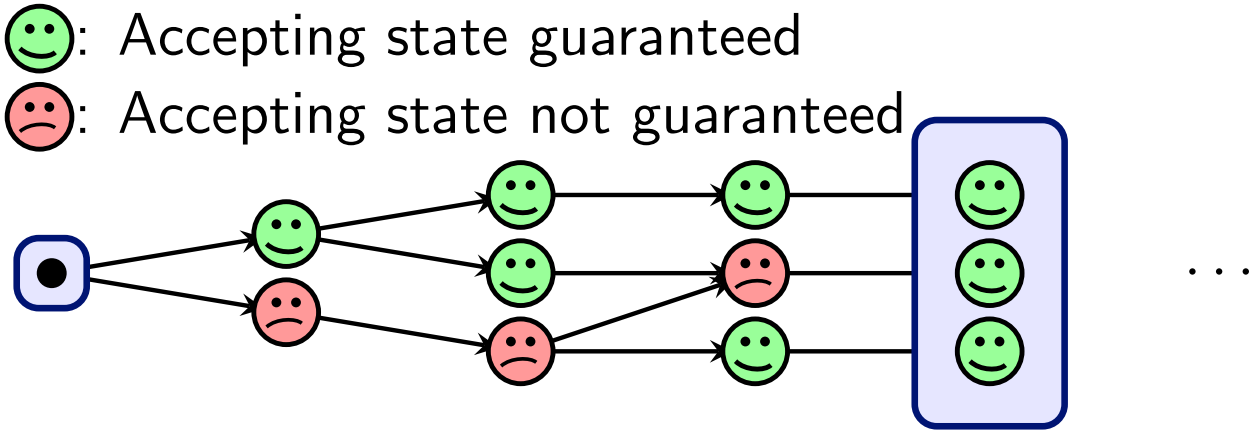


Breakpoint: All paths since last breakpoint visit accepting state

Breakpoint Construction



Breakpoint: All paths since last breakpoint visit accepting state

Lemma (Miyano and Hayashi, 1984)

A run of an alternating automaton is accepting if there exists a breakpoint sequence over it.

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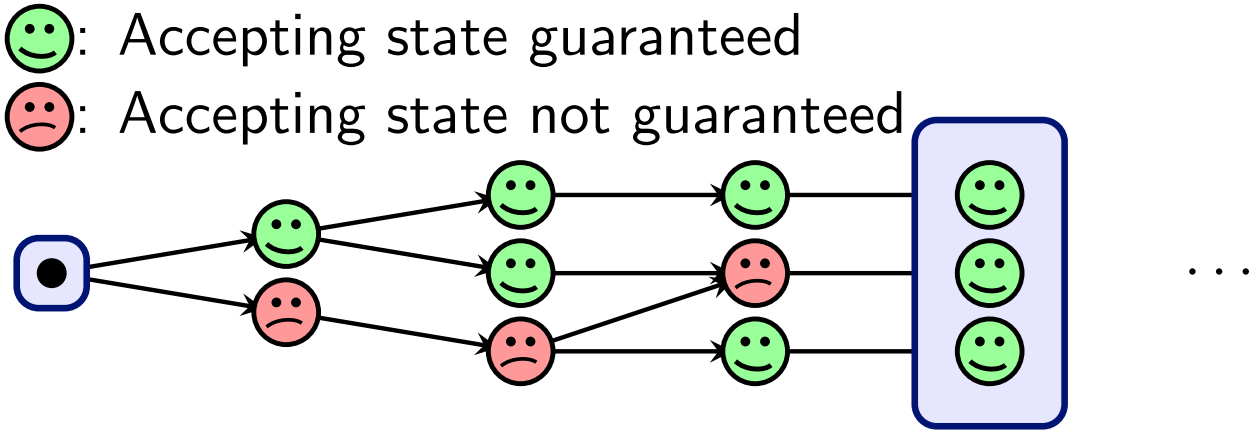

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A run of a $1 - \mathrm{AJA}$ *is accepting if there exists a breakpoint sequence over it.*

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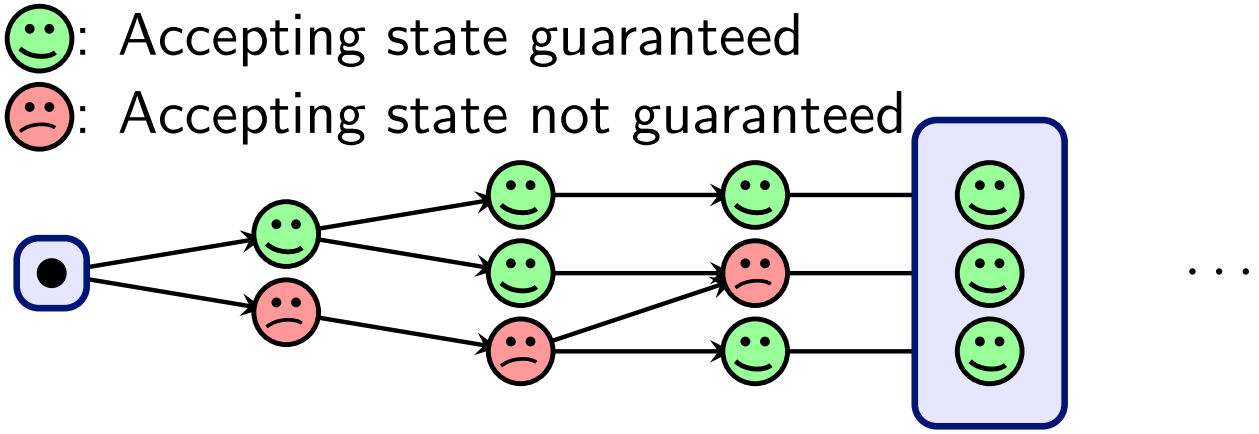

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Accepting States

- Keep track of 🙂 and 😕 states in state space
- Update 🙂 and 😕 states on the fly
- Accept and restart if all states are 🙂

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Accepting States ✓

Constructing a Tree Automaton

Component	Technique	
States	Subset Construction	✓
Transitions	Guess/Verify	✓
Accepting States		

Constructing a Tree Automaton

Component	Technique	
States	Subset Construction	✓
Transitions	Guess/Verify	✓
Accepting States	Breakpoint Construction	

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Accepting States	Breakpoint Construction	✓

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Component	Technique	
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Accepting States	Breakpoint Construction	✓

Theorem

For every VLDL formula φ we can construct a tree automaton $\mathfrak{T}$ of exponential size that recognizes the same language.

Guiding Questions

1. What are $1-\mathrm{AJAs}$? ✓
2. How to translate $1-\mathrm{AJAs}$ into tree automata?
   - How to translate words into trees? ✓

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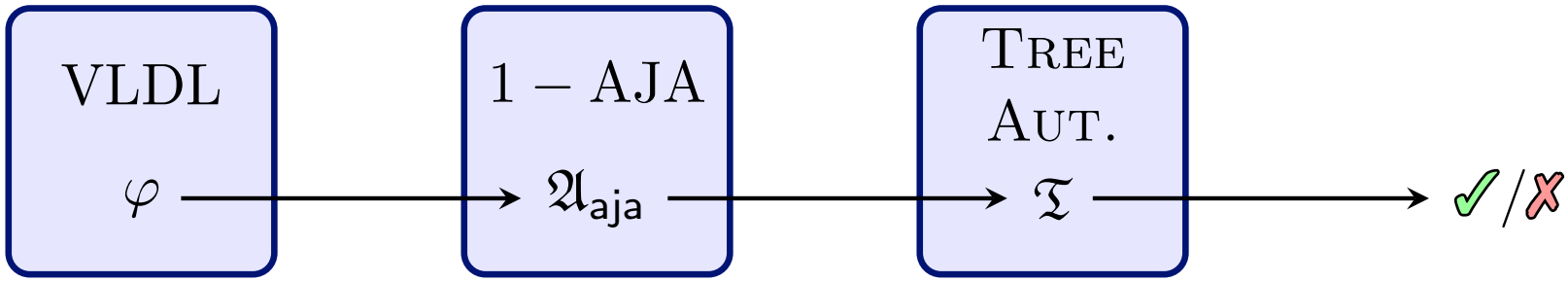
New Approach



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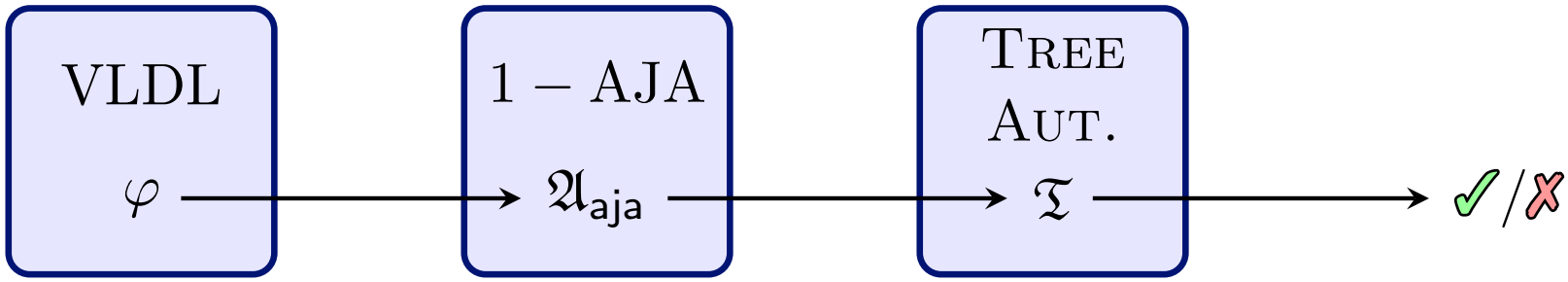
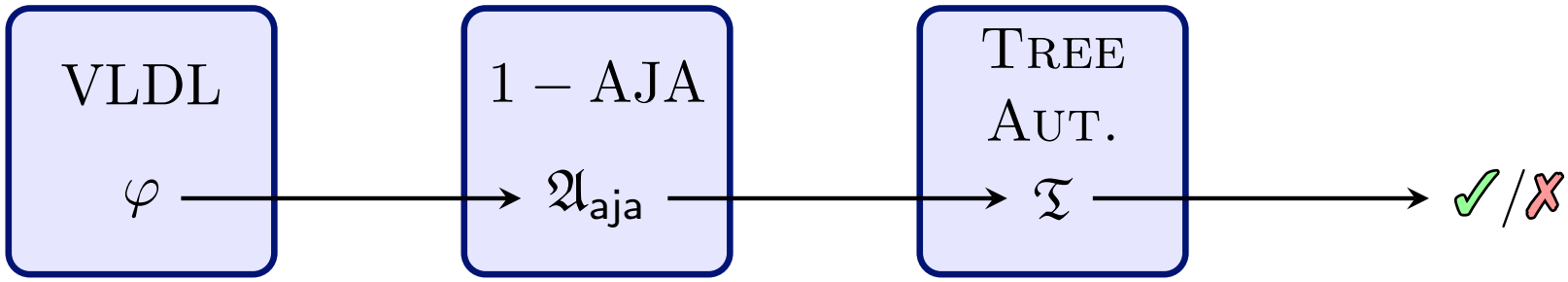


Lemma

The following problem is in PTime*: "Given a tree automaton* $\mathfrak{T}$*, does* $\mathfrak{T}$ *recognize the empty language?"*

New Approach



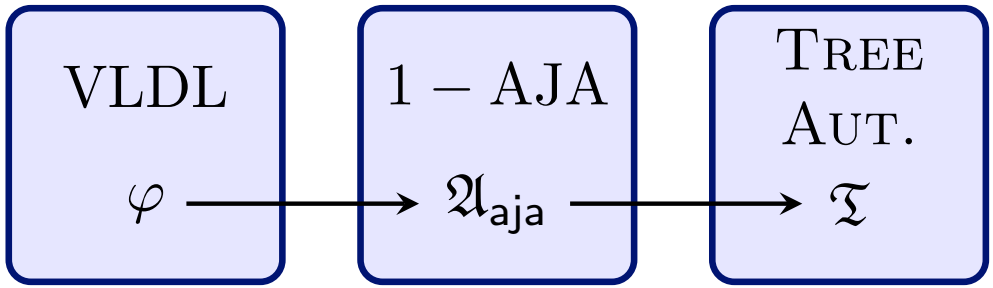
Lemma

The following problem is in PTime*: "Given a tree automaton* $\mathfrak{T}$*, does* $\mathfrak{T}$ *recognize the empty language?"*

Theorem

The following problem is in ExpTime*: "Given a* VLDL *formula* φ*, does* φ *define the empty language?"*

Model Checking



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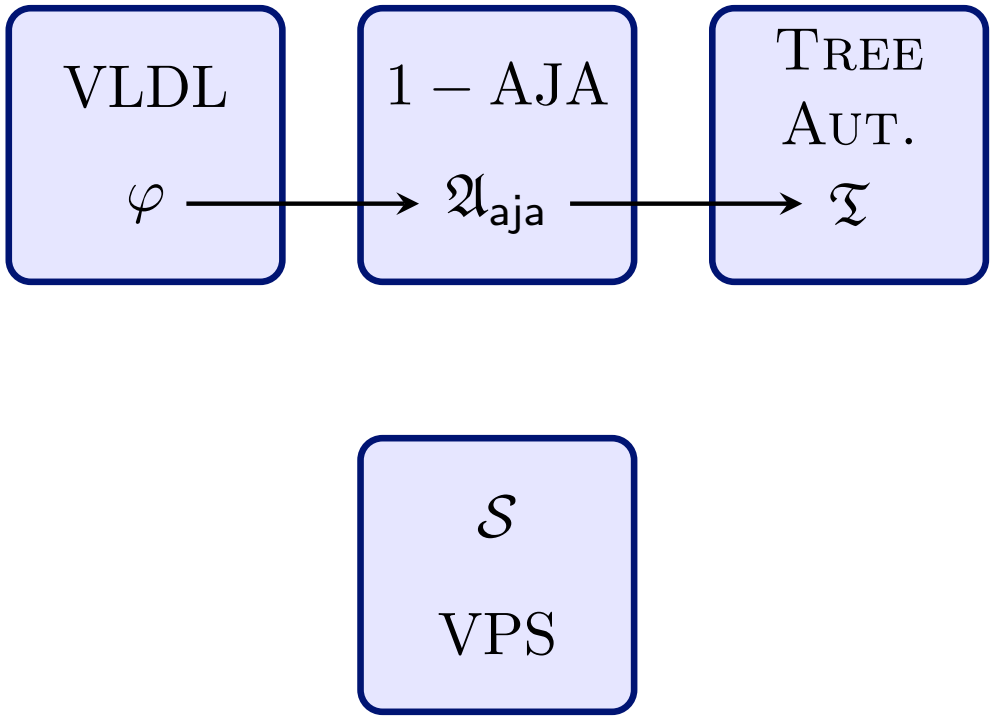

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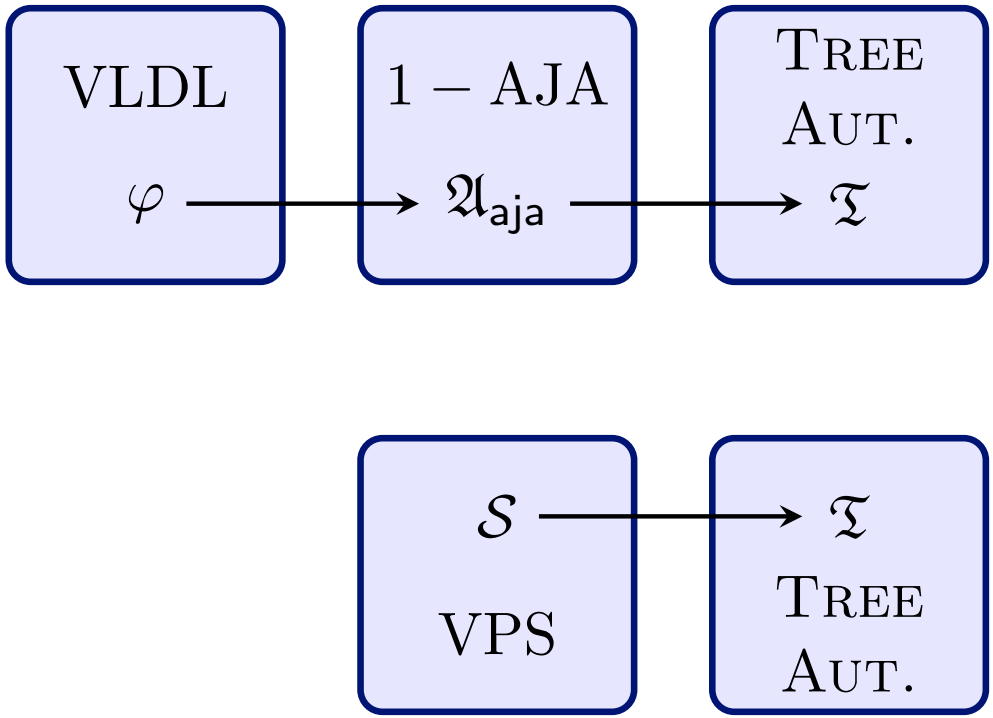
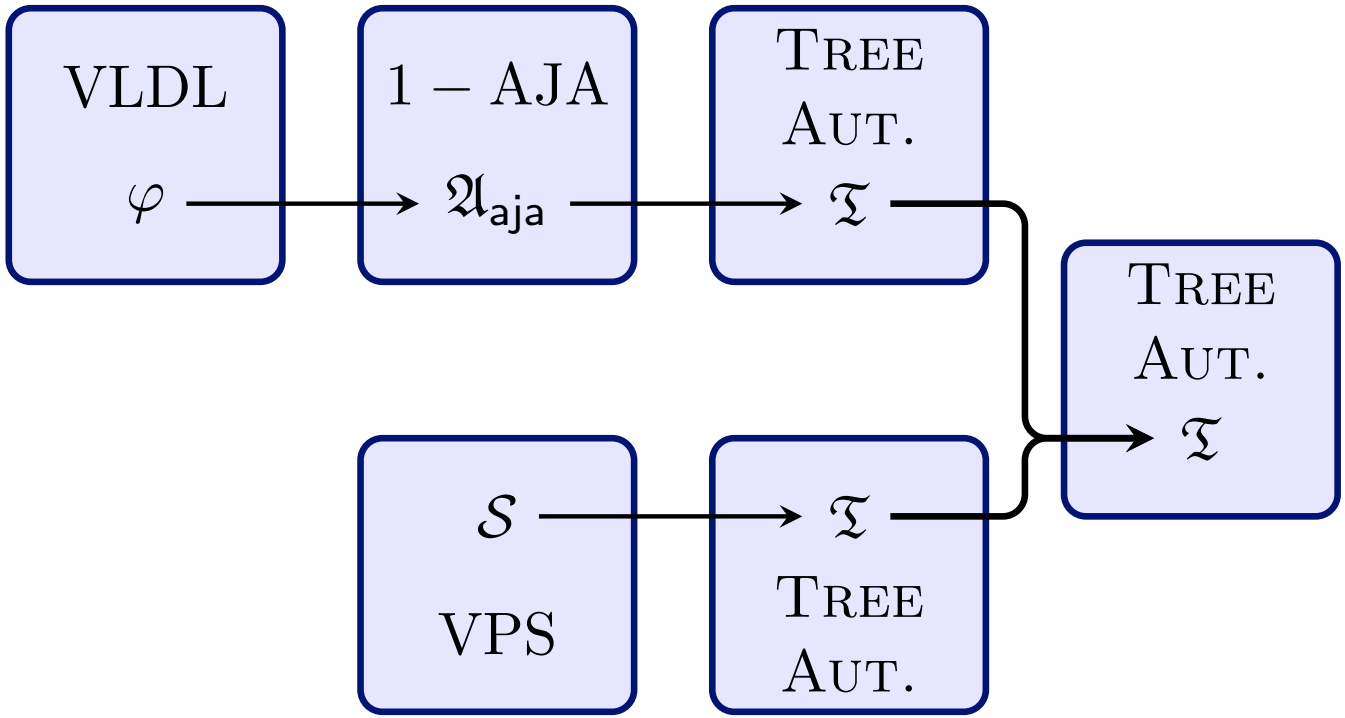
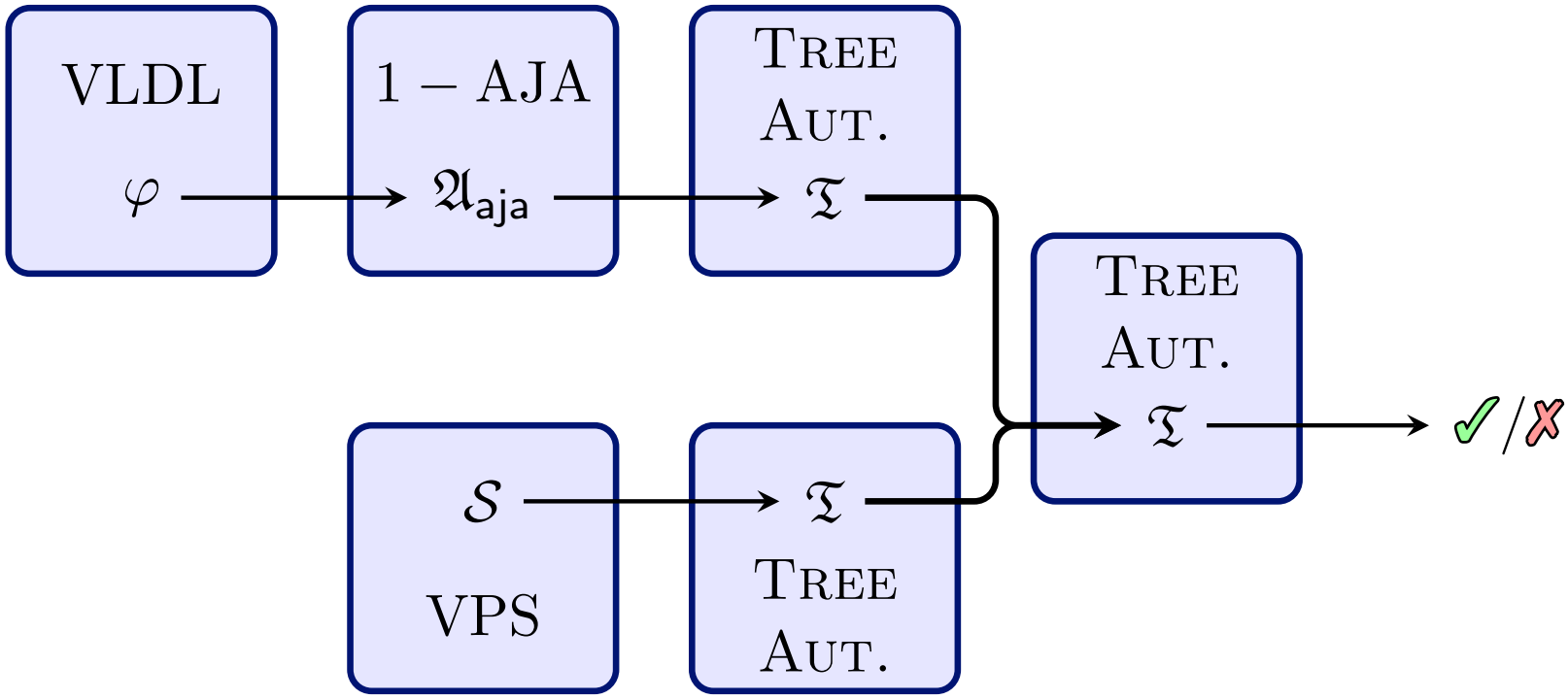


Model Checking



Model Checking



Theorem

The following problem is in ExpTime*: "Given a* VLDL *formula* φ *and a visibly pushdown system* $\mathcal{S}$*, do all traces of* $\mathcal{S}$ *satisfy* φ*?"*

Conclusion

- Connection between visibly pushdown words and stack trees
- Breakpoint technique is very versatile
- Putting VLDL on solid algorithmic foundation of Büchi games

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Future Work

- Games with VLDL winning conditions
- Prototypical Implementation, Comparison