Easy to Win, Hard to Master:
Playing Infinite Games Optimally

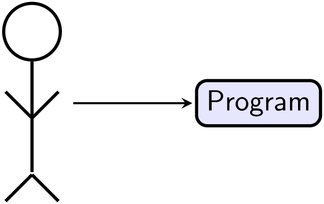
Alexander Weinert

Saarland University

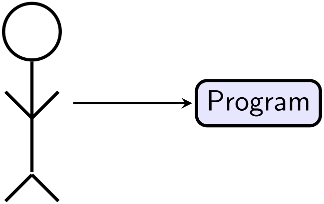
April 26th, 2017

Thesis Proposal Talk

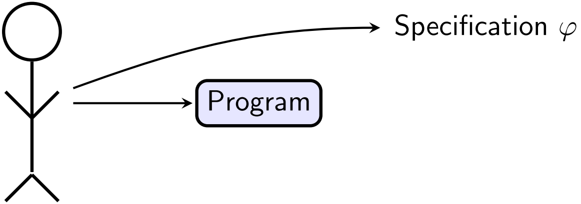
Programming



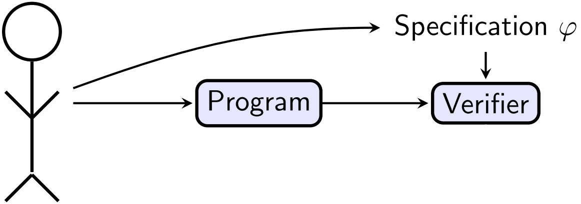
Program Verification



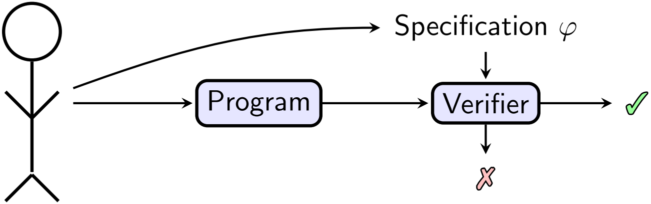
Program Verification



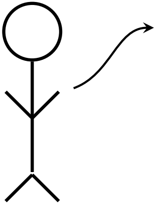
Program Verification



Program Verification

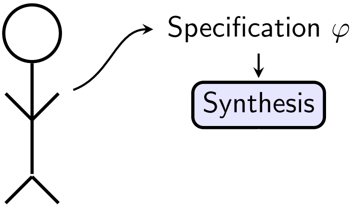


Program Synthesis

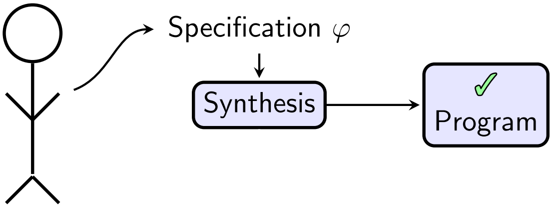


Specification φ

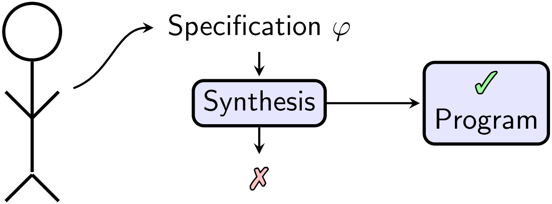
Program Synthesis



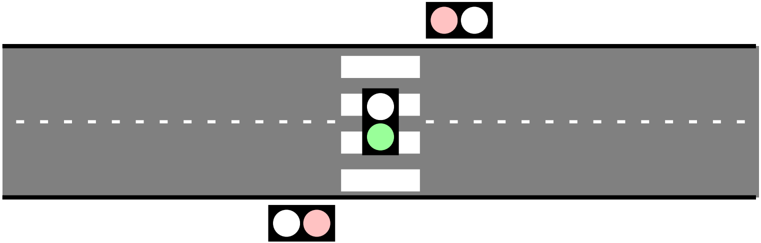
Program Synthesis



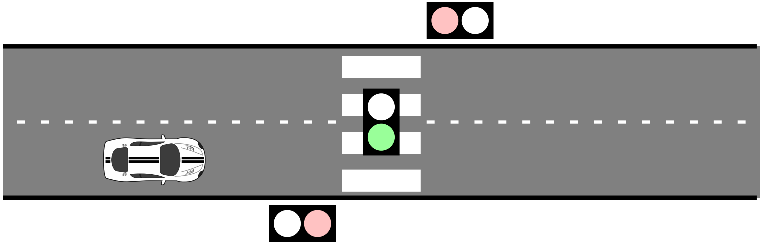
Program Synthesis



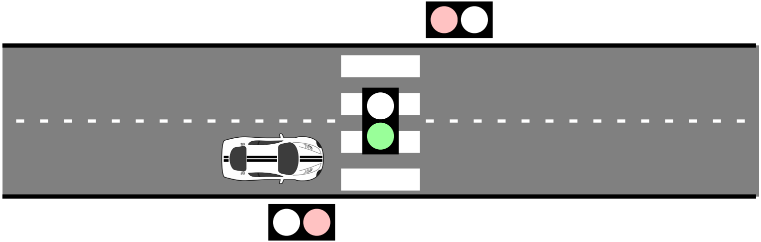
Example



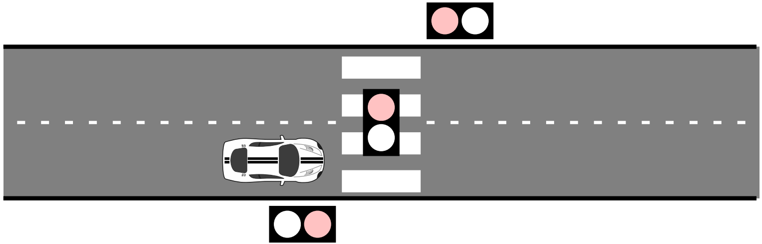
Example



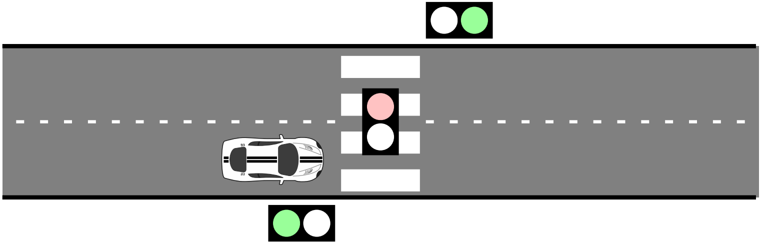
Example



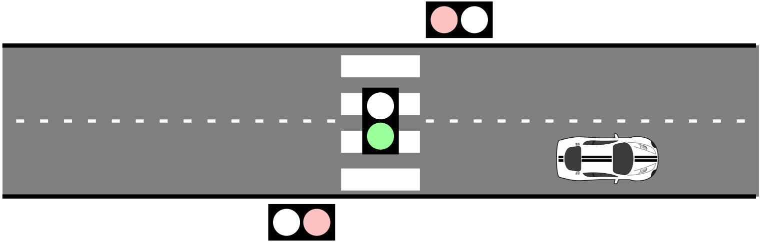
Example



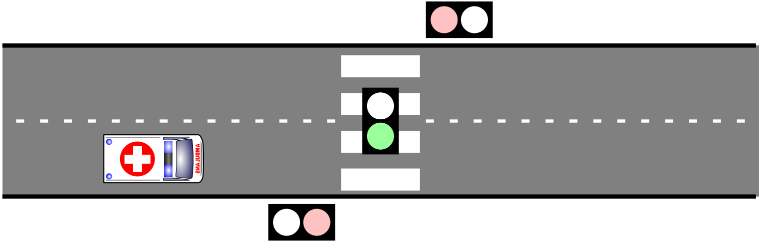
Example



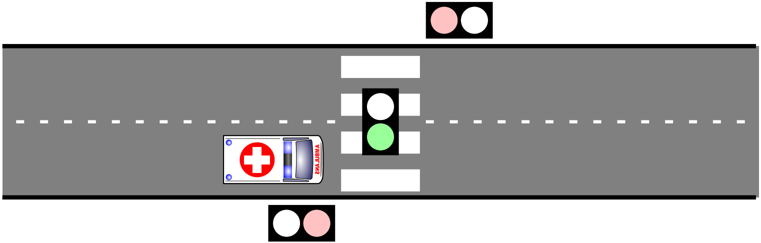
Example



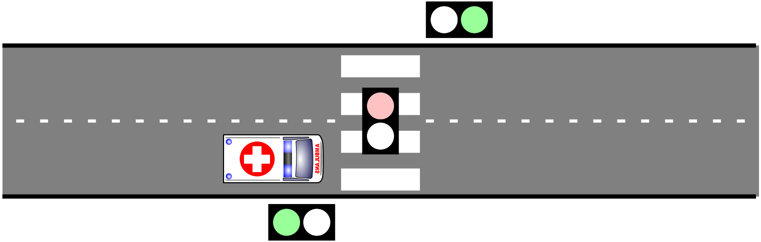
Example



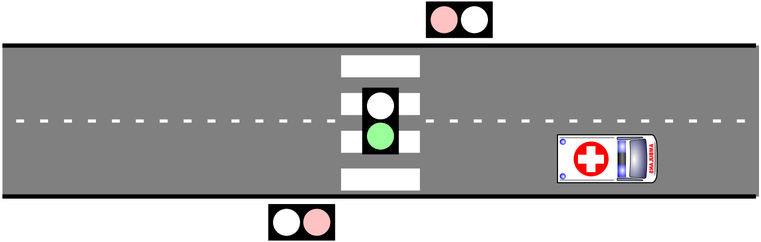
Example



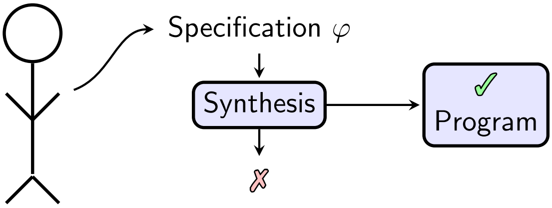
Example



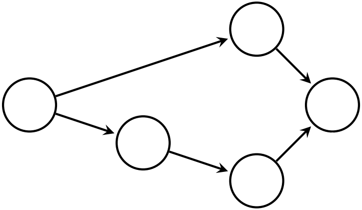
Example



Program Synthesis

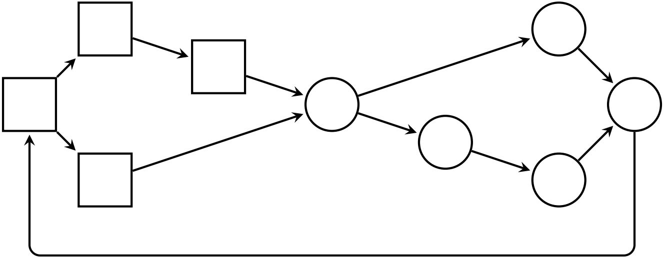


Parity Games



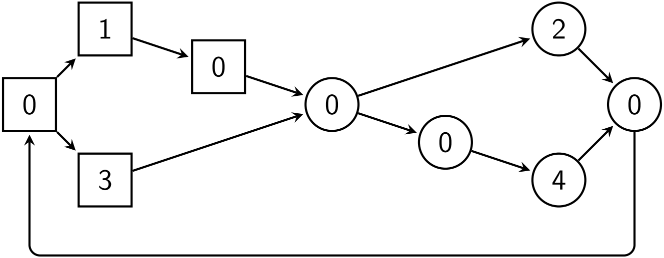
Example due to (Fijalkow and Chatterjee, Infinite-state games, 2013)

Parity Games



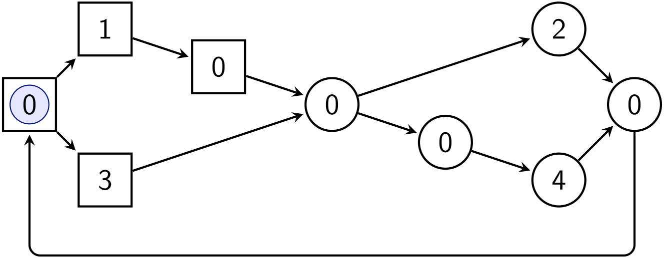
Example due to (Fijalkow and Chatterjee, Infinite-state games, 2013)

Parity Games



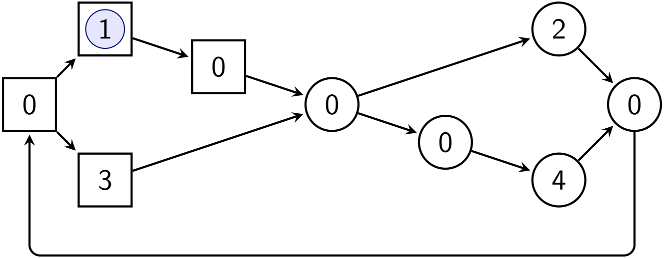
Example due to (Fijalkow and Chatterjee, Infinite-state games, 2013)

Parity Games



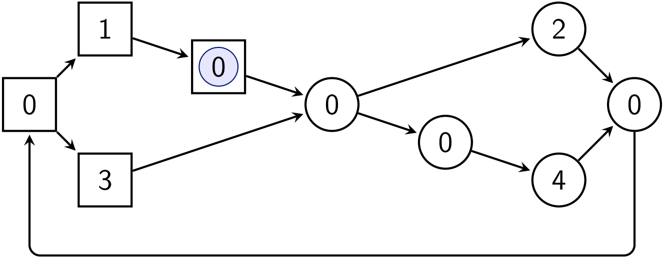
0

Parity Games



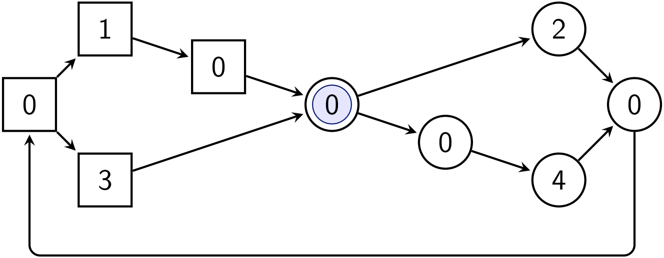
$0 \longrightarrow 1$

Parity Games



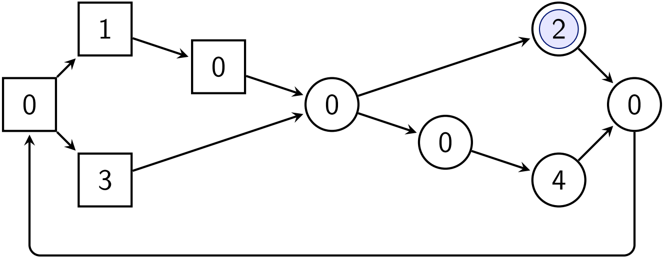
$0 \longrightarrow 1 \longrightarrow 0$

Parity Games



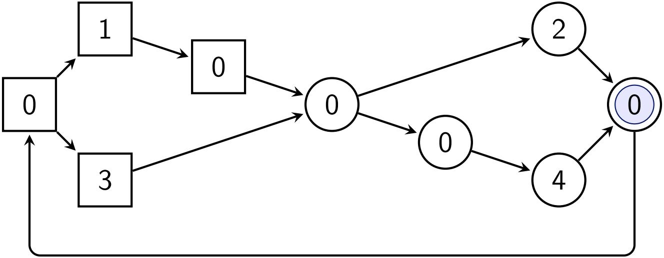
$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0$

Parity Games



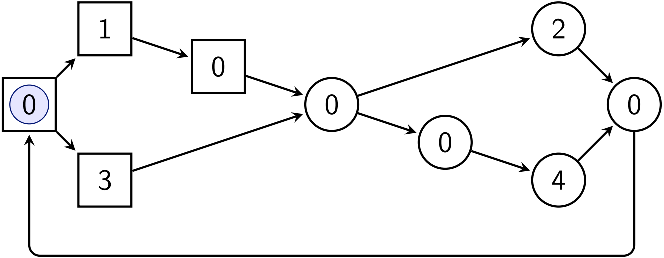
$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 2$

Parity Games



$$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 2 \longrightarrow 0$$

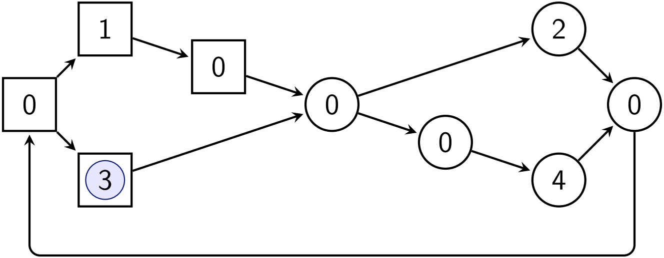
Parity Games



$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 2 \longrightarrow 0$

0

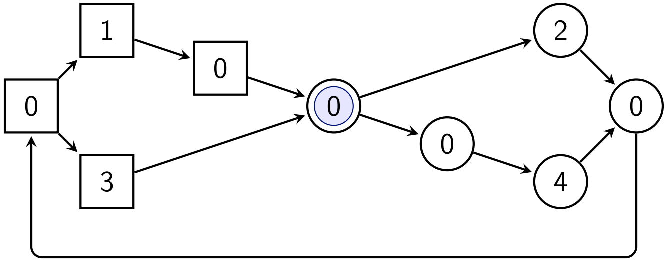
Parity Games



$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 2 \longrightarrow 0$

$0 \longrightarrow 3$

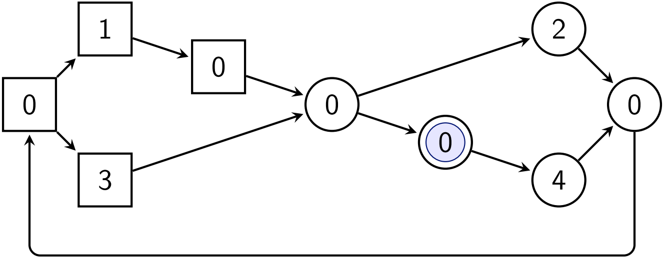
Parity Games



$$0 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 2 \rightarrow 0$$

$$0 \rightarrow 3 \rightarrow 0$$

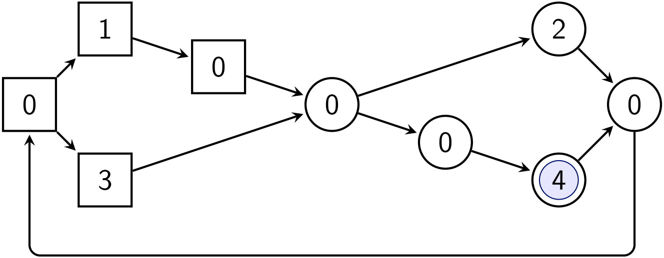
Parity Games



$$0 \to 1 \to 0 \to 0 \to 2 \to 0$$

$$\to 0 \to 3 \to 0 \to 0$$

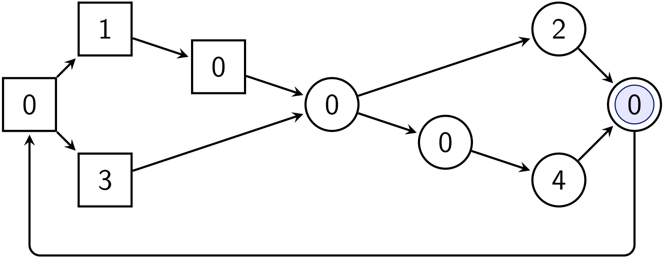
Parity Games



$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 2 \longrightarrow 0$

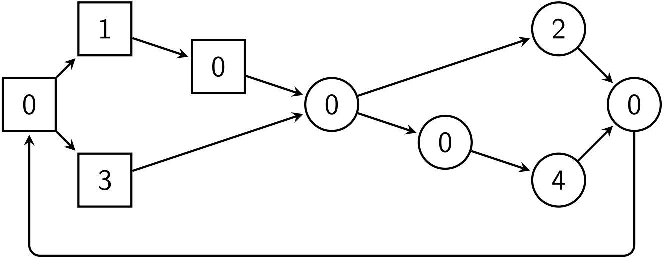
$0 \longrightarrow 3 \longrightarrow 0 \longrightarrow 0 \longrightarrow 4$

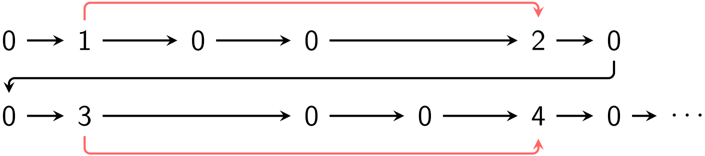
Parity Games



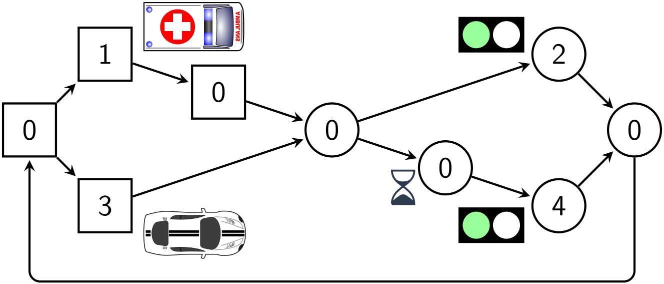
$$
\begin{array}{l}
0 \to 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 2 \to 0 \\
0 \to 3 \longrightarrow 0 \longrightarrow 0 \longrightarrow 4 \to 0 \to \cdots
\end{array}
$$

Parity Games





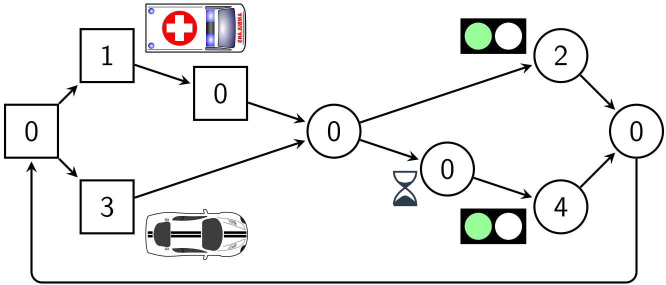
Parity Games



$$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 2 \longrightarrow 0$$

$$0 \longrightarrow 3 \longrightarrow 0 \longrightarrow 0 \longrightarrow 4 \longrightarrow 0 \longrightarrow \cdots$$

Parity Games

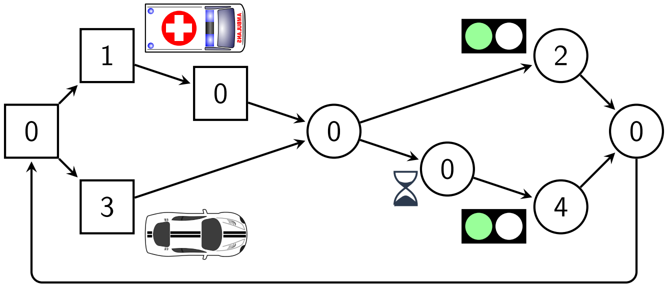


$$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 2 \longrightarrow 0$$

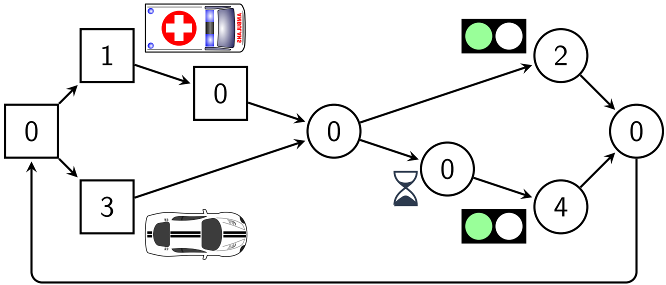
$$0 \longrightarrow 3 \longrightarrow 0 \longrightarrow 0 \longrightarrow 4 \longrightarrow 0 \longrightarrow \cdots$$

- Deciding winner in $\mathrm{NP} \cap \textsc{co}\text{-}\mathrm{NP}$
- Positional Strategies

Finitary Parity Games



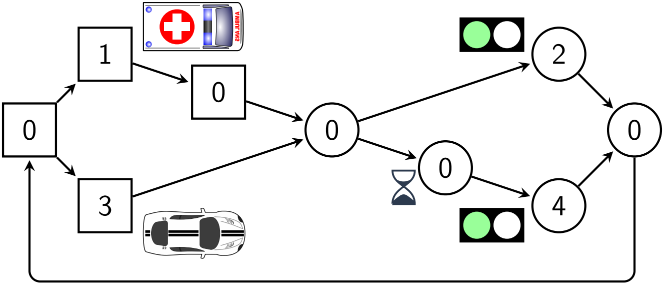
Finitary Parity Games



$$0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 2 \longrightarrow 0$$

$$0 \longrightarrow 3 \longrightarrow 0 \longrightarrow 0 \longrightarrow 4 \longrightarrow 0 \longrightarrow \cdots$$

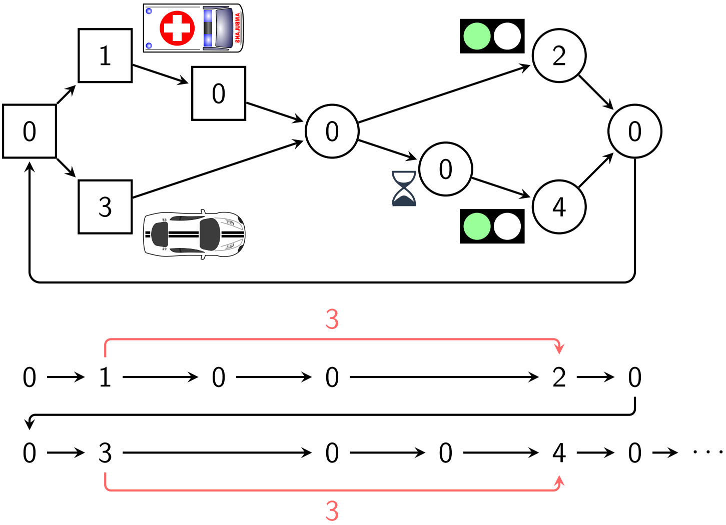
Finitary Parity Games



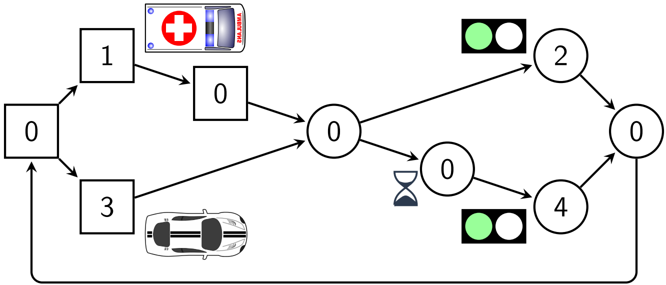
$$0 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 2 \rightarrow 0$$

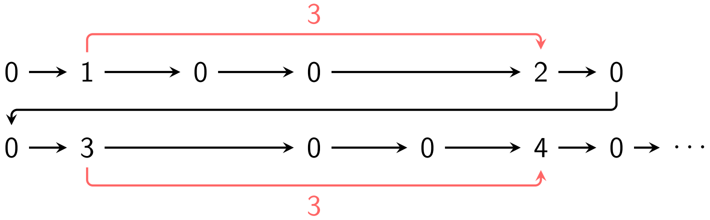
$$0 \rightarrow 3 \rightarrow 0 \rightarrow 0 \rightarrow 4 \rightarrow 0 \rightarrow \cdots$$

Finitary Parity Games



Finitary Parity Games





Goal for Player 0: Bound response times

Decision Problem

Theorem (Chatterjee, Henzinger, Horn, 2009)

The following decision problem is in $\textsc{PTime}$:

Input: *Finitary parity game* $\mathcal{G}$

Question: *Does there exist a strategy* σ *with* $\mathrm{Cst}(\sigma) < \infty$?

Decision Problem

Theorem (Chatterjee, Henzinger, Horn, 2009)

The following decision problem is in $\mathrm{PT{\scriptsize IME}}$*:*

Input: *Finitary parity game* $\mathcal{G}$

Question: *Does there exist a strategy* σ *with* $\mathrm{Cst}(\sigma) < \infty$*?*

Theorem (W., Zimmermann, 2016)

The following decision problem is $\mathrm{PS{\scriptsize PACE}}$*-complete:*

Input: *Finitary parity game* $\mathcal{G}$*, bound* $b \in \mathbb{N}$

Question: *Does there exist a strategy* σ *with* $\mathrm{Cst}(\sigma) \leq b$*?*

Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSpace-membership

Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSpace-membership

Necessity:

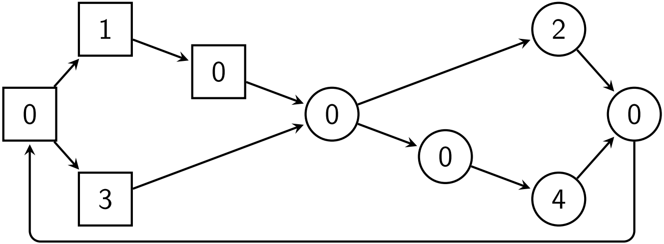
Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSPACE-membership

Necessity:



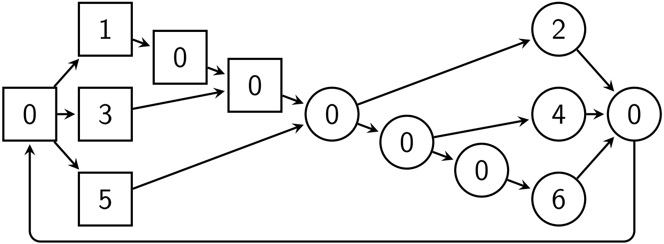
Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSPACE-membership

Necessity:



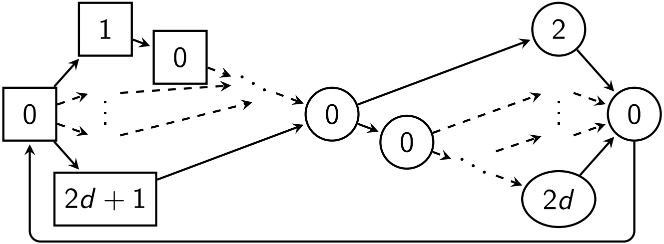
Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSPACE-membership

Necessity:



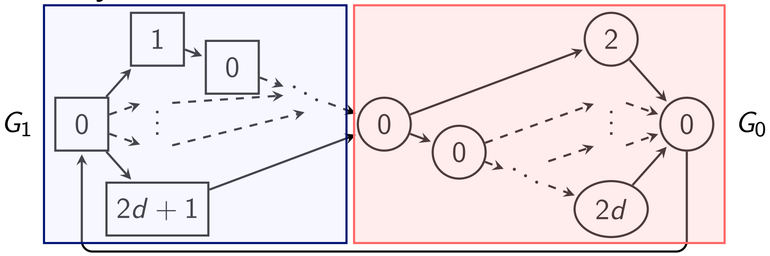
Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSPACE-membership

Necessity:



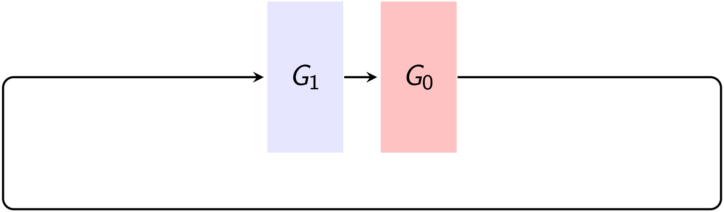
Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSPACE-membership

Necessity:



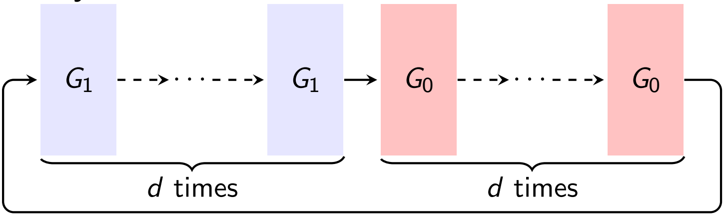
Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSPACE-membership

Necessity:



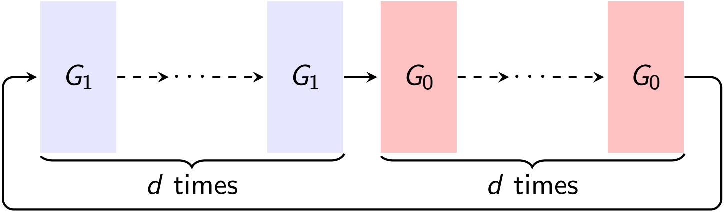
Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSPACE-membership

Necessity:



Player 0 needs to recall d positions with d possible values

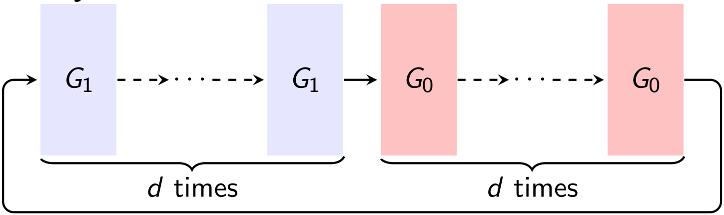
Memory Requirements (for Player 0)

Theorem (W., Zimmermann, 2016)

Optimal strategies for finitary parity games need exponential memory

Sufficiency: Corollary of proof of PSPACE-membership

Necessity:



Player 0 needs to recall d positions with d possible values

$\Rightarrow$ Player 0 requires $\approx 2^d$ many memory states

Results so far

	Parity
Complexity	$\mathrm{NP} \cap \textsc{co}\text{-}\mathrm{NP}$
Strategy Size	1

Results so far

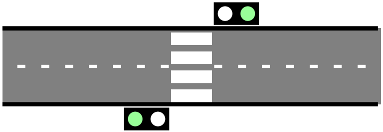
	Parity	Finitary Parity
		Winning
Complexity	$\mathrm{NP} \cap \textsc{co}\text{-}\mathrm{NP}$	$\textsc{PTime}$
Strategy Size	1	1

Results so far

	Parity	Finitary Parity	
		Winning	Optimal
Complexity	$\mathrm{NP} \cap \textsc{co}\text{-}\mathrm{NP}$	$\mathrm{P}\textsc{Time}$	$\mathrm{P}\textsc{Space}$-comp.
Strategy Size	1	1	Exp.

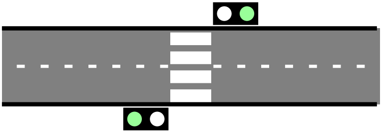
Outlook

So far

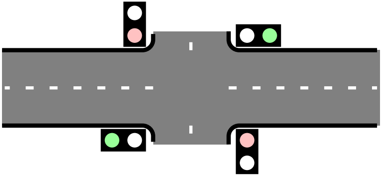


Outlook

So far

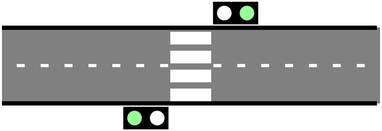


Multi-Dimensional Games

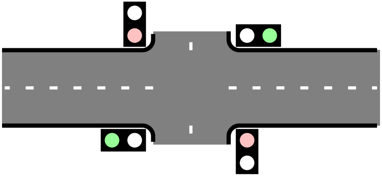


Outlook

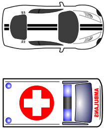
So far



Multi-Dimensional Games

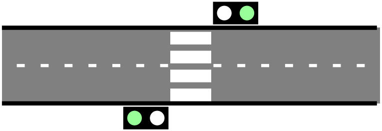


Imperfect Information

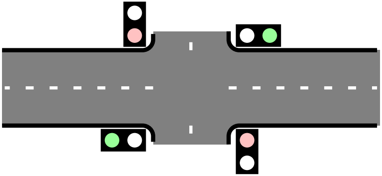


Outlook

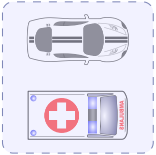
So far



Multi-Dimensional Games



Imperfect Information



Conclusion

Results so far: Forcing Player 0 to answer quickly in (finitary) parity games makes it harder

- to decide whether she can satisfy the bound
- for her to play the game

Conclusion

Results so far: Forcing Player 0 to answer quickly in (finitary) parity games makes it harder

- to decide whether she can satisfy the bound
- for her to play the game

Guiding Question: What costs does playing games optimally incur

- in terms of computing a strategy?
- in terms of the complexity of strategies?